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Which is demonstrated

From its First PRINCIPLES.

Wherein

Every Thing that is *Useful* and *Curious* in that ART is fully considered and explained.

Particularly

Four new and very concise Methods to determine the Areas of Right-lined Figures Arithmetically, or by Calculation, as well as the Geometrical ones heretofore treated of; with two other new Geometrical Methods much more accurate and ready than any of the former, never before made public.

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The whole illustrated with Copper-Plates.

THE THIRD EDITION.

By ROBERT GIBSON, *Teacher of the Mathematics.*

DUBLIN: Printed for *Laurence Flin*, at the Bible, in *Castle-Street*; and *Richard Fitzsimons*, at the *King's-Head*, in *High-Street*. MDCCLXVIII.

TO

THE RIGHT HONOURABLE

THOMAS CARTER, Esq;

MASTER of the ROLLS, &c. &c.

SIR,

THE great Knowledge You have acquired in all kinds of Polite Literature, and the Encouragement You constanly give for the Propagation of them in general, but more particularly to the Mathematical Sciences, are Motives sufficient to direct me to dedicate this Work to you.

Panegyrick, SIR, is not my Province, and if it were, I know I must offend one of your extensive Learning, should I attempt to display it on this Occasion: Yet I must say, that if but one fourth Part of the Nobility and Gentlemen of opulent Fortunes in this

Kingdom, were of your unlimited generous Diſpoſition in encouraging Arts and Sciences; we might in a few Years, be able to vie with any Country whatſoever.

That you may long enjoy an uninterupted State of good Health is the Wiſh of moſt; but of none more particularly, than of

SIR,

Your moſt obliged,
moſt obedient,
and moſt aſſured
Humble Servant,

ROBERT GIBSON.

PREFACE.

THE Word *Geometry* imports no more than *to measure the Earth*, or to measure Land; yet in a larger and more proper Sense, it is applied to all Sorts of Dimensions. It is generally supposed to have had its Rise among the *Egyptians*, from the River *Nile's* destroying and confounding all their Land-marks by its annual Inundations, which laid them under the Necessity of inventing certain Methods and Measures, to enable them to distinguish and adjust the Limits of their respective Grounds, when the Waters were withdrawn. And this Opinion is not entirely to be rejected, when we consider that *Moses* is said to have acquired this Art, when we resided at the *Egyptian* Court. And *Achilles Tatius* in the Beginning of his Introduction to *Aratus*'s *Phænomena*, informs us, that the *Egyptians* were the first who measured the Heavens and the Earth (and of course the Earth first) and that their Science in this Matter, was engraven on Columns, and by that means delivered to Posterity.

It is a Matter of some Wonder, that though Surveying appears to have been the first, or at least one of the first of the Mathematical Sciences, that the rest have met with much greater Improvements from the Pens of the most eminent Mathematicians,

maticians, while this ſeems to have been neglected; inſomuch that I have not been able to meet with one Author, who has ſufficiently explained the whole Art in its Theory and Practice: For the moſt part, it has been treated of in a practical Manner only; and the few who have undertaken the Theory, have in a great Meaſure omitted the Practice.

Theſe Conſiderations induced me to attempt a methodical, eaſy and clear Courſe of *Surveying*; how far I have ſucceeded in it, muſt be determined by the impartial Reader: The Steps I have taken to render the whole evident and familiar are as follow:

In Section the firſt, you have Decimal Fractions, the Square Root, Geometrical Definitions, ſome neceſſary Theorems and Problems; with the Nature and Uſe of the Tables of Logarithm Numbers, Sines, Tangents, and Secants.

The ſecond Section contains Plane Trigonometry right angled and oblique, with its Application in determining the Meaſures of inacceſſible Heights and Diſtances.

The third Section gives an Account of the Chains and Meaſures uſed in *Great-Britain* and *Ireland* Methods of Surveying and of taking inacceſſible Diſtances by the Chain only, with ſome neceſſary Problems; alſo a particular Deſcription of the ſeveral Inſtruments uſed in Surveying, with their reſpective Uſes.

The fourth Section contains five various Methods of finding the Areas of Maps, from their Geometrical

trical Conſtruction; two of which more conciſe than the reſt, were never before made public.

The fifth Section contains four new, and much more conciſe Methods of determining the Areas of Surveys from the Field Notes, or by Calculation than any hitherto publiſhed; and I venture to aſſert that it is impoſſible (from the Nature of right-lined Figures) that any Method or Methods more conciſe than theſe can be inveſtigated.

To theſe Methods is annexed a ſhort Table of Difference of Latitude and Half Departure, to every Degree and Quarter of a Degree of the Quadrant, the ſtationary Diſtance being one Chain; which will be found as ready, by a little Practice, and perhaps more exact, than thoſe already publiſhed. To this is annexed a Table for reducing Degrees of the *Circumferentor* to thoſe of the *Quarter Compaſs*, and the contrary; alſo the Method of changing Angles of the Field, taken by *Theodolite*, *Semicircle*, or *Plane Table*, to thoſe from the Meridian; for the greater Readineſs and Accuracy in Protraction, as well as to prepare them for Calculation.

Truth calls upon me to acknowledge, that the Methods by Calculation, herein ſet forth, got their Riſe from thoſe of the late *Thomas Burgh*, Eſq; who firſt diſcovered an univerſal Method for determining the Areas of right-lined Figures, and for which he obtained a parliamentary Reward. I hope therefore it cannot be conſtrued as an Intention in me to take from his great Merit, when I ſay, that the Methods herein contained are much more conciſe and ready than his.

Section

Section the ſixth contains the Nature of Off-ſets, and the Method of caſting them up by the Pen; The Nature and application of Interſections: The Methods of enlarging, diminiſhing, and connecting of Maps: The Method of tracing defaced Mearings from the Down (or any other) Surveys: The Variation of the Compaſs by Amplitudes and Azimuths, with ſome of its Uſes; to which is added, a Table of the Sun's Declination: The Method of reducing one Meaſure to another; how to find by what Scale a Map is laid down, having the Map and Area given: How to find the Content of Ground that is ſurveyed by a Chain that is too long or too ſhort: The Method of dividing Lands: And the whole concludes with ſome neceſſary Directions and Remarks on Surveys in general.

THE

THE PRINCIPLES OF SURVEYING.

SECT. I.

Containing Decimal Fractions, the Square Root, Geometrical Definitions, Theorems and Problems; with the Nature and Use of the Tables of Logarithm Numbers, Sines, Tangents, and Secants.

DEFINITION.

SURVEYING is that Art which enables us to give a Plan, or juſt Repreſentation, of any Piece or Parcel of Land, and to determine the Content thereof, in ſuch Meaſure as is agreeable and cuſtomary to the Country or Place where the Land is.

This Science depends on ſome Part of the Mathematics, which muſt be known before we can treat of it, wherefore we ſhall begin with

DECIMAL FRACTIONS.

If we ſuppoſe Unity or any one Thing to be divided into any aſſigned Number of equal Parts, this Number is called the Denominator; and if we chuſe to take any Number of ſuch Parts leſs than the Whole, this is called the Numerator of a Fraction.

The Numerator, in the Vulgar Form, is always wrote over the Denominator, and theſe are ſeparated by a ſmall Line thus $\frac{5}{12}$ or $\frac{7}{12}$ $\frac{\text{Numerator}}{\text{Denominator}}$ the firſt of theſe is called 5 Twelfths, and the latter 7 Twelfths of an Inch, Yard, Perch, &c. or of whatever the whole Thing originally was.

Fractions are expreſſed in two Forms, that is, either vulgarly or decimally.

All Fractions whoſe Denominators do not conſiſt of a Cypher or Cyphers ſet after Unity, are called vulgar ones, and their Denominators are always wrote under their Numerators. The treating of theſe would be foreign to our preſent Purpoſe. But Fractions whoſe Denominators conſiſt of an Unit, prefixed to one or more Cyphers, are called Decimal Fractions; the Numerators of which are wrote without their Denominators, and are diſtinguiſhed from Integers by a Point prefixed: Thus $\frac{2}{10}$ $\frac{42}{100}$ and $\frac{172}{1000}$ in the Decimal Form, are expreſſed by .2 .42 .172.

The Denominators of ſuch Fractions always conſiſting of an Unit, prefixed to as many Cyphers as there are Places of Figures in the Numerators, it follows, that any Number of Cyphers put after thoſe Numerators, will neither increaſe or leſſen their Value: For $\frac{3}{10}$ $\frac{30}{100}$ and $\frac{300}{1000}$ are all of the ſame Value, and will ſtand in the Decimal Form thus .3 .30 .300; but a Cypher or Cyphers prefixed to thoſe Numerators, leſſen their Value in a tenfold Proportion: For $\frac{3}{10}$ $\frac{03}{100}$ and $\frac{003}{1000}$ which in the Decimal Form we denote by .3 .03 and .003, are Fractions, of which the firſt is ten Times greater than the ſecond; and the ſecond, ten Times greater than the third.

Hence it appears, that as the Value and Denomination of any Figure or Number of Figures in common Arithmetic is enlarged, and becomes ten

or

or a hundred, or a thouſand Times greater, by placing one or two, or three Cyphers after it; ſo in decimal Arithmetic, the Value of any Figure or Number of Figures, decreaſes, and becomes ten, or a hundred, or a thouſand Times leſs, while the Denomination of it increaſes, and becomes ſo many Times greater, by prefixing one, or two, or three Cyphers to it: And that any Number of Cyphers, before an Integer, or after a decimal Fraction, have no Effect in changing their Values.

Integers.						Decimals.					
1	2	7	1	6	3	5	1	2	3	6	7
Hund. of Thouſ.	Tens of Thouſ.	Thouſands	Hundreds	Tens	Units	Tenths	Pts. of an Hund.	Pts. of a Thouſ.	Pts. of Ten Thouſ.	Pts. of Hun. Th.	Pts. of a Million

Addition of DECIMALS.

Having placed thoſe Figures which are equidiſtant from the Point, (as well Integers as Fractions) under each other, add them as if they were Integers.

EXAMPLES.

Add 4.7832 3.2543 7.8251 6.03 2.857 and 3.251 together. Place them thus,

$$\begin{array}{r} 4.7832 \\ 3.2543 \\ 7.8251 \\ 6.03 \\ 2.857 \\ 3.251 \\ \hline \end{array}$$

Anſwer 28.0006

Add 6.2 121.306 .75 2.7 and .0007 together.

121.306
.75
2.7
.0007
Anſwer 130.9567

What's the Sum of 6.57 1.026 .75 146.5 8.7 526 3.97, and .0271?

Anſwer 693.5431.

What's the Sum of 4.51 146.071 .507 .0006 132 62.71 .507 7.9 and .10712?

Anſwer 354.31272.

Subtraction of DECIMALS.

Having placed the Figures which are equidiſtant from the Point, under each other; deduct as if they were Integers.

EXAMPLE.

From 38.765 take 25.3741

25.3741
Anſwer 13.3909

From 2.4 take .8472

.8472
1.5528

From 71.45 take 8.4837248

Anſwer 62.9662752.

From

From 84 take 82.3412
Anſwer 1.6588

Multiplication of DECIMALS.

Place the Multiplicand, and Multiplier, after any Manner under each other; and having multiplied as in whole Numbers, cut off as many Places of Decimals in the Product, counting from the right Hand towards the left, as there are in the Multiplicand, and Multiplier: But if there be not a ſufficient Number of Places in the Product the Defect may be ſupplied, by prefixing Cyphers thereto.

For the Denominator of the Product, being an Unit, prefixed to as many Cyphers, as the Denominators of the Multiplier and Multiplicand contain of Cyphers, it follows, that the Places of Decimals in the Product, will be as many as in the Numbers from whence it aroſe.

EXAMPLES.

Multiply 48.765 by .003609
.003609

438885
292590
146295

Anſwer .175992885

Multiply .121
by .14

484
121

Anſwer .01694

Multiply

Multiply 121.6 by 2.76

```
        2.76
      ------
        7296
       8512
      2432
      ------
Answer 335.616
      ------
```

Multiply .0089789 by 1085
Answer 9.7421065

Multiply .248723 by .13587
Answer .03379399401

Division of DECIMALS.

Having divided as in whole Numbers, annexing Cyphers to the Dividend if they be wanted; the Decimal Places in the Divisor and Quotient must be equal to those in the Dividend, and the Defect supply'd by prefixing Cyphers to the Quotient.

For the Dividend is a Product, contained under the Divisor and Quotient; and that Product contains as many Places of Decimals as the Numbers do from whence it arose: Therefore the Difference between the Number of Decimals in the Dividend and Divisor, must be cut off in the Quotient.

EXAMPLES.

Divide .144 by .12

```
.12).144(1.2
     ----
      24
     ----
```

EXAM-

EXAMPLES.

Divide 63.72413456922 by 2718.

2718)63.72413456922(.02344522979

9364

12101

12293

14214

6245

8096

26609

21472

24462

Answer .02344522979

There being 11 decimal Figures in the Dividend, and none in the Divisor, 11 Figures are to be cut off in the Quotient; but as the Quotient itself consists of but 10 Figures, we prefix to them a Cypher to compleat that Number.

Divide 1.728 by .012

.012(1.728(144

52

48

Because

Becauſe the Number of decimal Figures in the Diviſor and Dividend, are alike, the Quotient will be Integers.

Divide 2.00000 by 3.1416

3.1416)2.00000(.636618

115040

207920

194240

57440

260240

8912

There being 4 decimal Figures in the Diviſor, and 10 including the Cyphers brought down in the Dividend, the Difference, which is 6 Figures, to be cut off in the Quotient.

Divide 87446071 by .004387.
Anſwer 199.33.
Divide .624672 by 482..
Anſwer .001296.
Divide 66.993548 by 27.4
Anſwer 2.44502.

PROB. I.

To Reduce a Vulgar Fraction to a Decimal one of the ſame Value.

Having annexed a ſufficient Number of Cyphers as Decimals, to the Numerator of the Vulgar Fraction, divide by the Denominator; and the Quotient thence ariſing, will be the decimal Fraction required.

EXAM-

EXAMPLES.

Reduce $\frac{3}{4}$ to a Decimal Fraction.

4)3.00(.75 Anſwer.

20

For $\frac{3}{4}$ of one Shilling, Yard, Perch, &c. is equal to one Fourth of three Shillings, Yards, Perches, &c. therefore if three be divided by 4, the Quotient will be the Anſwer.

Reduce $\frac{2}{5}$ to a Decimal Fraction.

5)2.0(.4 Anſwer.

Reduce $\frac{12}{25}$ to a Decimal Fraction.

25)12.00(.48 Anſwer.

200

Reduce $\frac{25}{218}$ to a Decimal Fraction.

Anſwer .1146789.

PROB. II.

To find the Value of a Decimal Fraction, in the known Parts of the Integer.

Multiply the Decimal propoſed into the Number of equal Parts contained in the Integer, and the Product will be the Number of ſuch Parts as are expreſſed by the Fraction.

What's the Value of .25 of a Pound Sterling?

20

Anſwer Shillings 5.00

C For

For 25 or $\frac{25}{100}$ of one Pound, is equal to the one hundredth Part of 25 Pounds, or of the Shillings in 25 Pounds, which are 500; therefore the one hundredth Part thereof will be 5 Shillings; which is effected by cutting off the two Cyphers, for the two Decimals, by a Point.

What's the Value of .385 of a Pound Sterling?

	.385
	20
Shillings	7.700
	12
Pence	8.400
	4
Farthings	1.600

What's the Value of .48 of a Chain of 50 Links?

	.48
	50
Anſwer, Links	24.00

What's the Value of .2864 of a Shilling?

	.2864
	12
Pence	3.4368
	4
Farthings	1.7472

What's the Value of .287 of a Pound Weight Troy?

	Oz.	dwt.	grs.
Anſwer	3.	8.	21.

What's

What's the Value of .2945 of a Pound Avoirdupoise?

	Oz.	dwts.
Answer	4	11 $\frac{1}{4}$

The EXTRACTION of the SQUARE ROOT.

A Square Number is the Product of a Number multiplied by itself; and the Number so multiplied is called the Root of that Square; thus 9 is the Square of 3, and 3 is the Root of 9, for 3 multiplied by 3 is 9.

If a Square Number be given to find its Root, observe if the Number of Figures or Places in the given Square be odd or even, if they be odd, find the Root of the first Figure, but if they be even, of the two first; under which place the Square of that Root, and deduct, placing the Root in the Quotient, and bring down two Figures to the Remainder.

Let the Double of the said Root be made a Divisor to all the Figures of that last Remainder, except the last; put the Quotient thereof with the Root, or former Quotient; and having multiplied it into the Numbers so formed, deduct the Product from the foregoing Figures or Resolvend: And in like Manner proceed, 'till all the Figures of the given Square are exhausted.

If there be any Decimals in the given Square, their Number must be even or made so, before we begin to find the Root, by adding a Cypher to the right Hand; and for every two Places of Decimals in the Square, let one be cut off in the Root.

EXAMPLES.

1. What's the Square Root of this Square Number 298116

```
    29,81,16(546
    25
    ----
 104)481
     416
    --------
1086).6516
      6516
    --------
      ....
```

Because the Number of Figures in the given Square Number is even, we find the nearest Square Number to the two first Figures 29, which is 25, the Root whereof 5, we set in the Quotient, and deduct 25 from 29, and to the Residue 4, we annex the following Figures 81, so we have 481 for a Resolvend.

The double of the first Figure in the Quotient being 10 is then set as a Divisor to 48, all the Figures in the Resolvend but the last; and finding it to be contained 4 Times, we annex the 4 to the Divisor and Quotient; the then Divisor 104 is multiplied by the last Figure in the Quotient 4, and the Product 416 is deducted from the Resolvend 481, to the Residue whereof is annexed the two following Figures in the Square, so we have 6516 for a new Resolvend, to all which Figures but the last we make 108, the double of 54, the Figures in the Quotient a Divisor, and finding it will be contained 6 Times, we place 6 in the Divisor and Quotient; the then Divisor 1086 is multiplied by the last Figure in the Quotient 6, and the Product being set under the Resolvend and thence

thence deducted leaves Nothing: So is 546 the Root sought.

For if the Root 546 be squared or multiplied by 546, the Product will be the square Number given.

2. What is the Square Root of 1710864?

```
               Root
  1,71,08,64(1308  Answer.
  1
  ---
23).71
    69
    ----
2608).20864
      20864
      -----
      .....
```

What is the Square Root of 3857.3?

Here being an odd decimal Figure, we annex any odd Number of Cyphers to make the Decimal Places even; and then extracting the Root as before, we thence cut off half the Number of Decimals that we have in the Square. Thus

```
                   Root
       3857.300000(62.107  Answer.
       36
       --
   122)257
       244
       ---
  1241)1330
       1241
       ----
124207) 890000
        869449
        ------
        .20551
```

If to the Square of this Root we add the remaining Figures 20551, we ſhall have our given Square, whoſe Root was required.

What is the Square Root of 16007.3104?
Anſwer 126.52.

What is the Square Root of 348.17320836?
Anſwer 18.6594.

What is the Square Root of 12345678987654321?
Anſwer 111111111.

The Application of this will hereafter be ſhewn.

THE

THE ELEMENTS OF Plane Geometry.

DEFINITIONS.

Plate I.

GEOMETRY is that Science wherein we consider the Properties of Magnitude.

2. A Point is that which has no Parts, being of itself indivisible, as A.

3. A Line has Length but no Breadth, as AB Figures 1 and 2.

4. The Extremities of a Line are Points, as the Extremities of the Line AB are the Points A and B. Figures 1 and 2.

5. A right Line is the shortest that can be drawn between any two Points, as the Line AB. Fig. 1. but if it be not the shortest, it is then called a curve Line, as AB. Fig. 2.

Plate I.

6. A Superficies or Surface is conſidered only as having Length and Breadth, without Thickneſs, as ABCD. Fig. 3.

7. The Extremities of a Superficies are Lines.

8. The Inclination of two Lines meeting one another (provided they do not make one continued Line) or the opening between them is called an Angle. Thus Fig. 4, the Inclination of the Line AB to the Line BC meeting each other in the Point B, or the opening of the two Lines BA and BC, is called an Angle, as ABC.

Note, When an Angle is expreſſed by three Letters, the Middle one is that at the angular Point.

10. When the Lines that form the Angle are Right ones, it is then called a Right-lined Angle, as ABC. Fig. 4. If one of them be Right and the other Curved, it is called a Mix'd-Angle, as B. Fig. 5. If both of them be curved it is called a Curved-lined or a Spherical Angle, as C. Fig. 6.

11. If a right Line CD (Fig. 7) fall upon another right Line AB, ſo as to incline to neither Side, but make the Angles ADC, CDB on each Side equal to each other, then thoſe Angles are called right Angles, and the Line CD a Perpendicular.

12. An obtuſe Angle is that which is wider or greater than a right one, as the Angle ADE Fig. 7. and an acute Angle is leſs than a right one, as EDB. Fig. 7.

13. Acute

Plate I.

13. Acute and obtuse Angles in general are called oblique Angles.

14. If a right Line CB (Fig. 8.) be fastened at the End C, and the other End B be carried quite round, then the Space comprehended is called a Circle; and the curve Line described by the Point B is called the Circumference or the Periphery of the Circle; the fixed Point C is called its Center.

15. The describing Line CB. (Fig. 8.) is called the Semidiameter or Radius, or any Line from the Center to the Circumference: Whence all Radii of the same or of equal Circles are equal.

16. The Diameter of a Circle is a right Line drawn thro' the Center, and terminating on either Side of the Circumference; and it divides the Circle and Circumference into two equal Parts called Semicircles; and is double the Radius, as AB or DE. Fig. 8.

18. The Circumference of every Circle is supposed to be divided into 360 equal Parts called Degrees, and each Degree into 60 equal Parts called Minutes, and each Minute into 60 equal Parts called Seconds, and these into Thirds, Fourths, &c. these Parts being greater or less as the Radius is.

19. A Chord is a right Line drawn from one End of an Arc, or Arch (that is any Part of the Circumference of a Circle) to the other; and is the Measure of the Arc. Thus the right Line HG is the Measure of the Arc HBG. Fig. 8.

20. The Segment of a Circle is any Part thereof, which is cut off by a Chord: Thus the Space which is comprehended between the Chord HG

 and

Plate I.

and the Arc HBG, or that which is comprehended between the said Chord HG and the Arc HDAEG are called Segments. Whence 'tis plain, Fig. 8.

1. That any Chord will divide the Circle into two Segments.

2. The less the Chord is, the more unequal are the Segments.

3. When the Chord is greatest it becomes a Diameter, and then the Segments are equal; and each Segment is a Semicircle.

21. A Sector of a Circle is a Part thereof less than a Semicircle which is contained between two Radii and an Arc: Thus the Space contained between the two Radii CH, CB, and the Arc HB is a Sector. Fig. 8.

22. The right Sine of an Arc, is a perpendicular Line let fall from one End thereof, to a Diameter drawn to the other End: Thus HL is the right Sine of the Arc HB.

The Sines on the same Diameter encrease 'till they come to the Center, and so become the Radius: Hence it is plain that the Radius CD is the greatest possible Sine, and thence is called the whole Sine.

Since the whole Sine CD (Fig. 8.) must be perpendicular to the Diameter (by Def. 22.) therefore producing DC to E the two Diameters AB and DE cross one another at right Angles, and thus the Periphery

Plate I.

Periphery is divided into four equal Parts, as BD, DA, AE, and EB; (by Def. 11.) and ſo BD becomes a Quadrant or the fourth Part of the Periphery: Therefore the Radius DC is always the Sine of a Quadrant, or of the fourth Part of the Circle BD.

Sines are ſaid to be of as many Degrees as the Arc contains Parts of 360: So the Radius being the Sine of a Quadrant becomes the Sine of 90 Degrees, or of the fourth Part of the Circle, which is 360 Degrees.

23. The verſed Sine of an Arc is that Part of the Diameter that lies between the right Sine and the Circumference: Thus LB is the verſed Sine of the Arc HB. Fig. 8.

24. The Tangent of an Arc is a right Line touching the Periphery, being perpendicular to the End of the Diameter, and is terminated by a Line drawn from the Center thro' the other End: Thus BK is the Tangent of the Arc HB. Fig. 8.

25. And the Line which terminates the Tangent that is CK, is called the Secant of the Arc HB. Fig. 8.

26. What an Arc wants of a Quadrant is called the Complement thereof: Thus DH is the Complement of the Arc HB.

27. And what an Arc wants of a Semicircle is called the Supplement thereof: Thus AH is the Supplement of the Arc HB. Fig. 8.

Plate I.

28. The Sine, Tangent, or Secant of the Complement of any Arc, is called the Co-Sine, Co-Tangent, or Co-Secant of the Arc itſelf: Thus FH is the Sine, DI the Tangent, and CI the Secant of the Arc DH; or they are the Co-Sine, Co-Tangent, or Co-Secant of the Arc HB. Fig. 8.

29. The Sine of the Supplement of an Arc, is the ſame with the Sine of the Arc itſelf, for drawing them according to Def. 22. there reſults the ſelf-ſame Line; thus HL is the Sine of the Arc HB, or of its Supplement ADH. Fig. 8.

30. The Meaſure of a right-lined Angle is the Arc of a Circle ſwept from the angular Point, and contained between the two Lines that form the Angle: Thus the Angle HCB (Fig. 8.) is meaſured by the Arc HB, and is ſaid to contain ſo many Degrees as the Arc HB does; ſo if the Arc HB is 60 Degrees, the Angle HCB is an Angle of 60 Degrees.

Hence Angles are greater or leſs according as the Arc deſcribed about the angular Point, and terminated by the two Legs, contain a greater or leſs Number of Degrees of the whole Circle.

31. The Sine, Tangent, and Secant of an Arc, is alſo the Sine, Tangent, and Secant of an Angle whoſe Meaſure the Arc is; thus becauſe the Arc HB is the Meaſure of the Angle HCB, and ſince HL is the Sine, BK the Tangent, and CK the Secant, BL the verſed Sine, HF the Co-Sine, DI the Co-Tangent, and CI the Co-Secant, &c. of the Arch BH; then HL is called the Sine, BK the

Tangent,

Plate I.

Tangent, CK the Secant, &c. of the Angle HCB, whose Measure is the Arc HB. Fig. 8.

32. Parallel Lines are such as are equidistant from each other, as AB, CD. Fig. 9.

33. A Figure is a Space bounded by a Line or Lines. If the Lines be right it is called a rectilineal Figure, if curved it is called a curvilineal Figure; but if they be partly right and partly curved Lines, it is called a mixt Figure.

34. The most simple rectilineal Figure is a Triangle, being composed of three right Lines, and is considered in a double Capacity; 1st, with respect to its Sides; and 2d, to its Angles.

35. In respect to its Sides it is either equilateral, having the three Sides equal, as A. Fig. 10.

36. Or Isosceles, having two equal Sides, as B. Fig. 11.

37. Or Scalene, having the three Sides unequal, as C. Fig. 12.

38. In respect to its Angles, it is either right-angled, having one right Angle, as D. Fig. 13.

39. Or obtuse angled, having one obtuse Angle, as E. Fig. 14.

40. Or acute angled, having all the Angles acute, as F. Fig. 15.

41. Acute

Plate I.

41. Acute and obtuſe angled Triangles are in general called oblique angled Triangles, in all which any Side may be called the Baſe, and the other two the Sides.

42. The perpendicular Height of a Triangle is a Line drawn from the Vertex to the Baſe perpendicularly: Thus if the Triangle ABC be propoſed, and BC be made its Baſe, then if from the Vertex A the Perpendicular AD be drawn to BC, the Line AD will be the Height of the Triangle ABC, ſtanding on BC as its Baſe. Fig. 16.

Hence all Triangles between the ſame Parallels have the ſame Height, ſince all the Perpendiculars are equal from the Nature of Parallels.

43. Any Figure of four Sides is called a quadrilateral Figure.

44. Quadrilateral Figures whoſe oppoſite Sides are parallel, are called Parallelograms: Thus ABCD is a Parallelogram. Fig. 3. 17. and A. B. Fig. 18. and 19.

45. A Parallelogram whoſe Sides are all equal and Angles right, is called a Square, as ABCD. Fig. 17.

46. A Parallelogram whoſe oppoſite Sides are equal and Angles right, is called a Rectangle or an Oblong, as ABCD. Fig. 3.

47. A Rhombus is a Parallelogram of equal Sides, and has its Angles oblique, as A. Fig. 18. and is an inclined Square.

48. A

Plate I.

48. A Rhomboides is a Parallelogram whose opposite Sides are equal and Angles oblique; as B. Fig. 19. and may be conceived as an inclined Rectangle.

49. Any quadrilateral Figure that is not a Parallelogram is call a Trapezium. Plate 7. Fig. 3.

50. Figures which consist of more than four Sides are called Polygons; if the Sides are all equal to each other they are called regular Polygons. They sometimes are named from the Number of their Sides, as a five-sided Figure is called a Pentagon, one of six Sides a Hexagon, &c. but if their Sides are not equal to each other, then they are called irregular Polygons, as an irregular Pentagon, Hexagon, &c.

51. Four Quantities are said to be in Proportion when the Product of the Extremes is equal to that of the Means: Thus if A multiplied by D, be equal to B multiplied by C, then A is said to be to B as C is to D.

POSTULATES or PETITIONS.

1. That a right Line may be drawn from any one given Point to another.

2. That a right Line may be produced or continued at Pleasure.

3. That from any Center and with any Radius the Circumference of a Circle may be described.

4. It is also required, that the Equality of Lines and Angles to others given, be granted as possible:

That

That it is poſſible for one right Line to be perpendicular to another, at a given Point or Diſtance; and that every Magnitude has its half, third, fourth, &c. Part.

Note, Though theſe Poſtulates are not always quoted, the Reader will eaſily perceive where and in what Senſe they are to be underſtood.

AXIOMS or Self-evident TRUTHS.

1. Things that are equal to one and the ſame Thing, are equal to each other.

2. Every Whole is greater than its Part.

3. Every Whole is equal to all its Parts taken together.

4. If to equal Things, equal Things be added, the Wholes will be equal.

5. If from equal Things, equal Things be deducted, the Remainders will be equal.

6. If to or from unequal Things, equal Things be added or taken, the Sums or Remainders will be unequal.

7. All right Angles are equal to one another.

8. If two right Lines not parallel, be produced towards their neareſt Diſtance, they will interſect each other.

9. Things which mutually agree with each other are equal.

NOTES

NOTES.

A Theorem is a Propoſition, wherein ſomething is propoſed to be demonſtrated.

A Problem is a Propoſition, wherein ſomething is to be done or effected.

A Lemma is ſome Demonſtration, previous and neceſſary, to render what follows the more eaſy.

A Corollary is a conſequent Truth, deduced from a foregoing Demonſtration.

A Scholium, is when Remarks or Obſervations are made upon ſomething going before.

The Signification of SIGNS.

The Sign =, denotes the Quantities between which it ſtands to be equal.

The Sign +, denotes the Quantity it precedes to be added.

The Sign ----, denotes that Quantity which it precedes to be ſubſtracted.

The Sign X, denotes the Quantities, between them to be multiplied into each other.

To denote that four Quantities, A, B, C, D, are proportional, they are uſually wrote thus, A : B : : C : D; and read thus A is to B, ſo is C to D; but when three Quantities A, B, C, are proportional the middle Quantity is repeated, and they are wrote A : B : : B : C.

GEOMETRICAL THEOREMS.

THEOREM I.

Plate I.

IF a right Line falls on another, as AB, or EB, does on CD, (*Fig.* 20.) *it either makes with it two right Angles, or two Angles equal to two right Angles.*

1. If AB be perpendicular to CD, then (by Def. 11.) the Angles CBA, and ABD, will be each a right Angle.

2. But if EB fall ſlantwiſe on CD, then are the Angles DBE + EBC = DBE + EBA (= DBA) + ABC, or to two right Angles. Q. E. D.

Corollary I. Whence if any Number of right Lines were drawn from one Point, on the ſame Side of a right Line; all the Angles made by theſe Lines will be equal to two right Angles.

2. And all the Angles which can be made about a Point, will be equal to four right Angles.

THEO-

THEO. II.

Plate I.

If one right Line crosses another, (as AC does BD) the opposite Angles made by those Lines, will be equal to each other. That is AEB to CED and BEC to AED. Fig. 21.

By Theorem I. BEC † CED = 2 right Angles.
and CED † DEA = 2 right Angles.

Therefore (by Axiom 1.) BEC†CED = CED† DEA : take CED from both, and there remains BEC=DEA. (by Axiom 5.) Q. E. D.

After the same Manner CED † AED = 2 right Angles; and AED†AEB= 2 right Angles; wherefore taking AED from both, there remains CED =AEB. Q. E. D.

THEO. III.

If a right Line crosses two Parallels, as GH does AB and CD. (Fig. 22.*) then*

1. *Their external Angles are equal to each other, that is GEB = CFH.*

2 *The alternate Angles will be equal, that is AEF = EFD and BEF = CFE.*

3. *The external Angle will be equal to the internal and opposite one on the same Side, that is GEB = EFD and AEG = CFE.*

Plate I.

4. *And the Sum of the internal Angles on the ſame Side, are equal to two right Angles; that is BEF† DFE are equal to two right Angles, and AEF † CFE are equal to two right Angles.*

1. Since AB is parallel to CD, they may be conſidered as one broad Line, croſſed by another Line, as GH; (then by the laſt Theo.) GEB = CFH, and AEG = HFD.

2. Alſo GEB = AEF, and CFH = EFD; but GEB = CFH (by Part 1. of this Theo.) therefore AEF = EFD. The ſame Way we prove FEB = EFC.

3. AEF = EFD; (by the laſt Part of this Theo.) but AEF = GEB (by Theo. 2.) Therefore GEB = EFD. The ſame Way we prove AEG = CFE.

4. For ſince GEB = EFD to both add FEB, then (by Axiom 4.) GEB†FEB = EFD†FEB, but GEB † FEB, are equal to two right Angles (by Theo. 1.) Therefore EFD † FEB are equal to two right Angles. After the ſame Manner we prove that AEF † CFE are equal to 2 Right Angles. Q. E. D.

THEO. IV.

In any Triangle ABC, one of its Legs as BC being produced towards D, it will make the external Angle ACD equal to the two internal oppoſite Angles taken together. Viz. to B and A. Fig. 23.

Thro'

Plate I.

Thro' C, let CE be drawn parallel to AB: then ſince BD cuts the two parallel Lines, BA, CE; the Angle ECD = B, (by Part 3 of the laſt Theo.) and again, ſince AC cuts the ſame Parallels the Angle ACE = A (by Part 2. of the laſt) Therefore ECD + ACE = ACD = B + A. Q. E. D.

THEO. V.

In any Triangle ABC, all the three Angles taken together are equal to two right Angles, viz. A + B + ACB = 2 *right Angles.* F. 23.

Produce BC to any Diſtance as D, then (by the laſt) ACD = B + A; to both add ACB; then ACD + ACB = A + B + ACB: But ACD + ACB = 2 right Angles (by Theo. 1.); therefore the three Angles A + B + ACB = 2 right Angles. Q. E. D.

Cor. 1. Hence if one Angle of a Triangle be known, the Sum of the other two is alſo known: For ſince the three Angles of every Triangle contain two right ones, or 180 Degrees, therefore 180 — the given Angle will be equal to the Sum of the other two; or 180 — the Sum of two given Angles, gives the other one.

Cor. 2. In every right-angled Triangle, the two acute Angles are = 90 Degrees, or to one right Angle: Therefore 90 — one acute Angle, gives the other.

THEO.

Plate I.

THEO. VI.

If any two Triangles, ABC, DEF, there be two Sides AB, AC, in the one, severally equal to DE, DF in the other, and the Angle A contained between the two Sides in the one, equal to D in the other; then the remaining Angles of the one, will be severally equal to those of the other, viz. B = E and C = F: And the Base of the one BC, will be equal to EF that of the other. Fig. 24.

If the Triangle ABC be supposed to be laid on the Triangle DEF, so as to make the Points A and B coincide with D and E, which they will do, because AB = DE (by the Hypothesis); and since the Angle A = D, the Line AC will fall along DF, and inasmuch as they are supposed equal, C will fall in F; seeing therefore the three Points of one coincide with those of the other Triangle, they are manifestly equal to each other; therefore the Angle B = E and C = F, and BC = EF. Q. E. D.

LEMMA.

If two Sides of a Triangle abc be equal to each other, that is ac=cb; the Angles which are opposite to those equal Sides will also be equal to each other, viz. a=b, Fig. 11.

For let the Triangle abc be divided into two Triangles acd, dcb, by making the Angle acd = dcb (by Postulate 4.) then because ac=bc, and cd, common, (by the last) the Triangle adc = dcb; and therefore the Angle a = b. Q. E. D.

Cor.

Plate I.

Cor. Hence if from any Point in a Perpendicular which biſects a given Line, there be drawn right Lines to the Extremities of the given one, they with it will form an Iſoſceles Triangle.

THEO. VII.

The Angle BCD at the Center of a Circle ABED is double the Angle BAD at the Circumference. Fig. 25.

Through the Point A, and the Center C, draw the Line ACE: Then the Angle ECD=CAD + CDA; (by Theo. 4) but ſince AC = CD being Radii of the ſame Circle, it is plain (by the preceding Lemma) that the Angles ſubtended by them will be alſo equal, and that their Sum is double to either of them, that is DAC+ADC is double to CAD, and therefore ECD is double to CAD; after the ſame Manner BCE is double to CAB, wherefore, BCE + ECD or BCD is double to BAC + CAD or to BAD. Q. E. D.

Cor. 1. Hence an Angle at the Circumference is meaſured by half the Arc it ſubtends or ſtands on.

Cor. 2. Hence all Angles at the Circumference of a Circle which ſtands on the ſame Chord as AB, are equal to each other, for they are all meaſured by half the Arc they ſtand on, *viz.* by half the Arc AB. Fig. 26.

Cor.

Plate I.

Cor. 3. Hence an Angle in a Segment greater than a Semi-circle is less than a right Angle; thus ADB is measured by half the Arc AB, but as the Arc AB is less than a Semicircle, therefore half the Arc AB, or the Angle ADB is less than half a Semicircle, and consequently less than a Right Angle. Fig. 26.

Cor. 4. An Angle in a Segment less than a Semicircle is greater than a right Angle, for since the Arc AEC is greater than a Semicircle, its half, which is the Measure of the Angle ABC, must be greater than half a Semicircle, that is greater than a right Angle. Fig. 27.

Cor. 5. An Angle in a Semicircle is a right Angle, for the Measure of the Angle ABD, is half of a Semicircle AED, and therefore a right Angle.

THEO. VIII.

If from the Center C of a Circle ABE, there be let fall the Perpendicular CD on the Chord AB, it will bisect it in the Point D. Fig. 29.

Let the Lines CA and CB be drawn from the Center to the Extremities of the Chord, then since CA=CB, the Angle CAB=CBA (by the Lemma). But the Triangles ADC, BDC are right angled ones, since the Line CD is a Perpendicular; and so the Angle ACD=DCB; (by Cor. 2. Theo. 5.) then have we AC, CD, and the Angle ACD in one Triangle; severally equal to CB, CD, and the Angle BCD in the other: Therefore (by Theo. 6) AD=DB. Q. E. D.

So

Plate I.

Cor. Hence it follows, that any Line biſecting a Chord at right Angles is a Diameter; for a Line drawn from the Center perpendicular to a Chord biſects that Chord at Right Angles; therefore converſely a Line biſecting a Chord at right Angles muſt paſs thro' the Center, and conſequently be a Diameter.

THEO. IX.

If from the Center of a Circle ABE there be drawn a Perpendicular CD on the Chord AB, and produced till it meets the Circle in F, that Line CF. will biſect the Arc AB in the Point F. Fig. 29.

Let the Lines AF and BF be drawn, then in the Triangles ADF, BDF; AD=BD (by the laſt); DF is common, and the Angle ADF=BDF being both right, for CD or DF is a Perpendicular. Therefore (by Theo. 6.); AF=FB; but in the ſame Circle equal Lines are Chords of equal Arcs, ſince they meaſure them (by Def. 19.); whence the Arc AF = F B, and ſo AFB is biſected in F, by the Line CF.

Cor. Hence the Sine of an Arc is half the Chord of twice that Arc. For AD is the Sine of the Arc AF, (by Def. 22.) AF is half the Arc, and AD half the Chord AB (by Theo. 8.) Therefore the Cor. is plain.

THEO. X.

Plate I.

In any Triangle ABD, the half of each Side is the Sine of the opposite Angle. Fig. 30.

Let the Circle ADB be drawn thro' the Points A, B, D; then the Angle DAB is measured by half the Arc BKD, (by Cor. 1. Theo. 7.) *viz.* the Chord of BK is the Measure of the Angle BAD; therefore (by Cor. to the last) BE the half of BD is the Sine of BAD: The same Way may be proved, that half of AD is the Sine of ABD, and the half of AB the Sine of ADB. Q. E. D.

THEO. XI.

If a right Line GH cuts two other right Lines AB; CD, so as to make the alternate Angles AEF, EFD equal to each other, then the Lines AB and CD will be parallel. Fig. 22.

If it be denied that AB is parallel to CD, let IK be parallel to it; then IEF=(EFD)=AEF (by Part 2. Theo. 3.) a greater to a less which is absurd, whence IK is not parallel; and the like we can prove of all other Lines but AB; therefore AB is parallel to CD. Q. E. D.

THEO. XII.

If two equal and parallel Lines AB, CD, be joined by two other Lines AD, BC, those shall be also equal and parallel. Fig. 3.

Let

Plate I.

Let the Diameter or Diagonal BD be drawn, and we will have the two Triangles ABD, CDB; whereof AB in one is = to CD in the other, DB common to both, and the Angle ABD = CDB (by Part 2. Theo. 3.); therefore (by Theo. 6.) AD = CB, and the Angle CBD = ADB, and thence the Lines AD and BC are parallel, by the preceding Theorem.

Cor. 1. Hence the Quadrilateral Figure ABCD is a Parallelogram, and the Diagonal BD bisects the same, inasmuch as the Triangle ABD = BDC, as now proved.

Cor. 2. Hence also the Triangle ADB on the same Base AB, and between the same Parallel with the Parallelogram, ABCD, is half the Parallelogram.

Cor. 3. It is hence also plain, that the opposite Sides of a Parallelogram are equal; for it has been proved that ABCD being a Parallelogram, AB will be = CD and AD = BC.

THEO. XIII.

All Parallelograms on the same or equal Bases and between the same Parallels are equal to one another, that is if BD = GH, and the Lines BH and AF parallel, then the Parallelogram ABDC = BDFE = EFHG. Fig. 31.

For AC (= DB = EF (by Cor. the last); to both add CE, then AE = CF. In the Triangles ABE, CDF; AB = CD and AE = CF and the Angle BAE = DCF (by Part 3. Theo. 3.); therefore the Triangle ABE = CDF. (by Theo. 6.) let the Triangle CKE be taken from both, and we will have

Plate I.

have the Trapezium ABKC = KDFE; to each of these add the Triangle BKD, then the Parallelogram ABCD = BDEF; in like Manner we may prove the Parallelogram EFHG = BDEF. Wherefore ABDC = BDEF = EFHG. Q. E. D.

Cor. Hence it is plain that Triangles on the same or equal Bases, and between the same Parallels, are equal, seeing (by Cor. 2. Theo. 12.) they are the Halves of their respective Parallelograms.

THEO. XIV.

In every right-angled Triangle, ABC, the Square of the Hypothenuse or longest Side, BC, or BCMH, is equal to the Sum or the Squares made on the other two Sides AB and AC, that is to ABDE and ACGF. (*Fig.* 32.)

Thro' A draw AKL perpendicular to the Hypothenuse BC, join AH, AM, DC and BG; in the Triangles BDC, ABH, BD = BA being Sides of the same Square, and also BC = BH, and the included Angle DBC = ABH, (for DBA = CBH being both right, to both add ABC, then DBC = ABH) therefore the Triangle DBC = ABH (by Theo. 6.) but the Triangle DBC is half of the Square ABDE (by Cor. 2. Theo. 12) and the Triangle ABH is half the Parallelogram BKLH (by the same); therefore half the Square ABDE is equal to half the Parallelogram BKLH, and the Square ABDE equal to the Parallelogram BKLH. The same Way it may be proved, that the Square ACGF, is equal to the Parallelogram KCLM. So ABDE + ACGF the Sum of the

Squares,

Plate I.

Squares, = BKLH+KCML, the Sum of the two Parallelograms or Square BCMH; therefore the Sum of the Squares on AB and AC is equal to the Square on BC. Q. E. D.

Cor. 1. Hence the Hypothenuſe of a right-angled Triangle may be found by having the Legs; thus, the Square Root of the Sum of the Squares of the Baſe and Perpendicular, will be the Hypothenuſe.

Cor. 2. Having the Hypothenuſe and one Leg given to find the other; the Square Root of the Difference of the Squares of the Hypothenuſe and given Leg, will be the required Leg.

THEO. XV.

In all Circles the Chord of 60 Degrees is always equal in Length to the Radius.

Thus in the Circle AEBD, if the Arc AEB be an Arc of 60 Degrees, and the Chord AB be drawn: then AB =CB=AC. (*Fig.* 33.)

In the Triangle ABC, the Angle ACB is 60 Degrees being meaſured by the Arc AEB; therefore the Sum of the other two Angles is 120 Degrees (by Cor. 1. Theo. 5.) but ſince AC=CB, the Angle CAB= CBA (by Lemma preceding Theo. 7.) conſequently each of them will be 60, the half of 120 Degrees, and the three Angles will be equal to one another, as well as the three Sides: Wherefore AB=BC=AC. Q. E. D

Cor.

Plate I.

Cor. Hence the Radius, from whence the Lines on any Scale is formed, is the Chord of 60 Degrees on the Line of Chords.

THEO. XVI.

If in two Triangles ABC, abc, all the Angles of one, be each respectively equal to all the Angles of the other, that is A=a, B=b, C=c: Then the Legs opposite to the equal Angles will be proportional, viz.

AB: ab: :AC: ac *Fig.* 34.

AB: ab: :BC: bc

and AC: ac: :BC: bc

For the Triangles being inscribed in two Circles, it is plain since the Angle A = a, the Arc BDC = bdc, and consequently the Chord BC is to bc, as the Radius of the Circle ABC is to the Radius of the Circle abc; (for the greater the Radius is, the greater is the Circle described by that Radius; and consequently the greater any particular Arc of that Circle is, so the Chord, Sine, Tangent, &c. of that Arc will be also greater. Therefore in general, the Chord, Sine, Tangent, &c. of any Arc is proportional to the Radius of the Circle); the same Way the Chord AB is to the Chord ab, in the same Proportion. So AB : ab : : BC : bc; the same Way the rest may be proved to be proportional.

THEO. XVII.

If from a Point A without a Circle DBCE there be drawn two Lines ADE, ABC, each of them cutting the Circle in two Points; the Product of one whole

Plate I.

whole Line into its external Part, viz. AC into AB, will be equal to that of the other Line into its external Part, viz. AE into AD. Fig. 35.

Let the Lines DC, BE be drawn, in the two Triangles ABE, ADC; the Angle AEB = ACD, (by Cor. 2. Theo. 7.) the Angle A is common, and (by Cor. 1. Theo. 5.) the Angle ADC = ABE; therefore the Triangles ABE, ADC, are mutually equiangular, and consequently, (by the last) AC: AE:: AD: AB; wherefore AC multiplied by AB, will be equal to AE multiplied by AD. Q. E. D.

THEO. XVIII.

Plate II. Fig. 1.

Triangles ABC, BCD, and Parallelograms ABCF, and BDEC, having the same Altitude, have the same Proportion between themselves as their Bases AB, and BD.

Let any aliquot Part of AB be taken, which will also measure BD: Suppose that to be Ag which will be contained twice in AB, and three Times in BD, the Parts Ag, gB, Bh, hi, and iD being all equal, and let the Lines gC, hC, and iC, be drawn: Then (by Cor. to Theo. 13.) all the small Triangles AgC, gCB, BCh, &c. will be equal to each other; and will be as many as the Parts into which their Bases were divided: Therefore it will be, as the Sum of the Parts in one Base, is to the Sum of those in the other, so will be the Sum of small Triangles in the first, to the Sum of the small Triangles in the second Triangle; that is AB: BD :: ABC: BDC.

Whence

Plate II.

Whence alſo the Parallelograms ABCF, and BDEC being (by Cor. 2. Theo. 12.) the Doubles of the Triangles, are likewiſe as their Baſes. Q. E. D.

Note. Wherever there are ſeveral Quantities connected with the Sign : : The Concluſion is always drawn from the firſt two, and laſt two Proportionals.

THEO. XIX.

Triangles ABC, DEF, ſtanding upon equal Baſes AB and DE, are to each other as their Altitudes CG and FH. Fig. 2.

Let BI be perpendicular to AB and equal to CG, in which let KB = FH, and let AI and AK be drawn.

The Triangle AIB = (ACB by Cor. to Theo. 13.) and AKB = DEF; but (by Theo. 18.) BI : BK : : ABI : ABK. That is CG : FH : : ABC : DEF. Q. E. D.

THEO. XX.

If a right Line BE be drawn parallel to one Side of a Triangle ACD, it will cut the two other Sides proportionally, viz. *AB : BC : : AE : ED. Fig.* 3.

Draw CE and BD; the Triangles BEC and EBD being on the ſame Baſe BE and under the ſame Parallel CD, will be equal (by Cor. to Theo. 13.) therefore (by Theo. 18.) AB : BC : : (BEA : BEC or BEA : BED) : : AE : ED. Q. E. D.

Cor.

Plate II.

Cor. 1. Hence alſo AC : AB : : AD : AE : For AC : AB : : (AEC : AEB : : ABD : AEB) : : AD : AE.

Cor. 2. It alſo appears that a right Line, which divides two Sides of a Triangle proportionally, muſt be parallel to the remaining Side.

Cor. 3. Hence alſo Theo. 16. is manifeſt; ſince the Sides of the Triangles ABE, ACD being equiangular, are proportional.

THEO. XXI.

If two Triangles ABC, ADE, have one Angle in BAC, equal to one Angle DAE, and the Sides about the equal Angles proportional, that is AB : AD : : AC : AE, then the Triangles will be mutually equiangular. Fig. 4.

In AB take Ad = AD, and let de, be parallel to BC, meeting AC in e.

Becauſe (by the firſt Cor. to the foregoing Theo.) AB : Ad : : (AD) AC : Ae, and (by the Hypotheſis, or what is given in the Theorem) AB : AD : AC : AE; therefore Ae = AE ſeeing AC bears the ſame Proportion to each; and (by Theo. 6.) the Triangle Ade = ADE, therefore the Angle Ade = D and Aed = E, but ſince ed and BC are parallel (by Part 3. Theo. 3.) Ade = B, and Aed = C, therefore B = D and C = E. Q. E. D.

Plate II.

THEO. XXII.

Equiangular Triangles ABC, DEF, are to one another in a Duplicate Proportion of their Homologous or like Sides; or as the Squares AK, and DM of their Homologous Sides. Fig. 5.

Let the Perpendiculars CG and FH be drawn, as well as the Diagonals BI and EL.

The Perpendiculars make the Triangles ACG and DFH equiangular, and therefore similar (by Theo. 16.) for becauſe the Angle CAG = FDH and the right Angle AGC = DHF, the remaining Angle ACG = DFH, (by Cor. 2. Theo. 5.)

Therefore GC : FH : : (AC : DF : :) AB : DE, or which is the ſame Thing GC : AB : : FH : DE, for FH multiplied by AB = AB multiplied by FH.

By Theo. 19 ABC : ABI : : (CG : AI, or AB as before : : FH DE, or DL : :) DFE : DLE, therefore ABC : ABI : : DFE : DLE, or ABC : AK : : DFE : DM, for AK is double the Triangle ABI, and DM double the Triangle DEF, by Cor. 2. Theo. 11. Q. E. D.

THEO. XXIII.

Like Polygons ABCDE, abcde, are in a Duplicate Proportion to that of the Sides AB, ab, which are between the equal Angles A and B, and a and b, or as the Squares of the Sides AB, ab. Fig. 6.

Draw AD, AC, ad, ac.

By

Plate II.

By the Hypothesis AB : ab : : BC : bc, and thereby also the Angle B = b; therefore (by Theo. 21.) BAC = bac; and ACB = acb; in like Manner EAD = ead, and EDA = eda. If therefore from the equal Angles A, and a, we take the equal ones EAD+BAC = ead+bac the remaining Angle DAC = dac, and if from the equal Angles D and d, EDA = eda be taken, we shall have ADC = adc: and in like Manner if from C and c be taken BCA = bca we shall have ACD = acd; and so the respective Angles in every Triangle will be equal to those in the other.

By Theo. 22. ABC : abc : : the Square of AC to the Square of ac, and also ADC : adc : : the Square of AC, to the Square of ac; therefore from Equality of Proportions ABC : abc : : ADC : adc, in like Manner we may shew that ADC : adc : EAD : ead: therefore it will be as one Antecedent, is to one Consequent; so are all the Antecedents, to all the Consequents. That is ABC : abc as the Sum of the three Triangles in the first Polygon, is to the Sum of those in the last. Or ABC will be to abc, as Polygon to Polygon.

The Proportion of ABC to abc (by the foregoing Theo.) is as the Square of AB is to the Square of ab, but the Proportion of Polygon to Polygon is as ABC to abc as now shewn: therefore the Proportion of Polygon to Polygon is as the Square of AB, to the Square of ab.

Plate I.

THEO. XXV.

Let DHB be a Quadrant of a Circle deſcribed by the Radius CB: HB an Arc of it, and DH its Complement; HL or FC the Sine, FH or CL its Co-Sine; BK its Tangent, DI its Co-tangent; CK its Secant, and CI its Co-ſecant. Fig. 8.

1. The Co-ſine of an Arc, is to the Sine, as Radius is to the Tangent.

2. Radius is to the Tangent of an Arc, as the Co-ſine of it is to the Sine.

3. The Sine of an Arc is to its Co-ſine, as Radius to its Co-tangent.

4. Or Radius is to the Co-tangent of an Arc, as its Sine to its Co-ſine.

5. The Co-tangent of an Arc is to Radius, as Radius to the Tangent.

6. The Co-Sine of an Arc is to Radius, as Radius is to the Secant.

7. The Sine of an Arc is to Radius, as the Tangent is to the Secant.

The Triangles CLH, and CBK being ſimilar, (by Theo. 16.)

1. CL : LH : : CB : BK.
2. Or, CB : BK : : CL : LH.

The

Plate I.

The Triangles CFH, and CDI, being similar.

3. CF (or LH): FH : : CD : DI.

4. CD : DI : : CF, (or LH) : FH.

The Triangles CDI and CBK are similar; for the Angle CID = KCB, being alternate ones (by Part 2. Theo. 3.) the Lines CB and DI being parallel: The Angle CDI = CBK being both right, and consequently the Angle DCI = CKB, wherefore,

5. DI : CD · : CB : BK.

And again, making Use of the similar Triangles CLH, and CBK.

6. CL : CB : : CH : CK.

7. HL : CH : BK : CK.

GEOMETRICAL PROBLEMS.

PROB. I.

Plate II.

TO make a Triangle of three given right Lines BO, LB, LO, of which any two must be greater than the third. Fig. 7.

Lay BL from B to L, from B with the Line BO, describe an Arc, and from L with LO describe another Arc; from O, the Intersection Point of those Arcs draw BO, and OL, and BOL is the Triangle required.

This is manifest from the Construction.

PROB. II.

At a Point B in a given Right Line BC, to make an Angle equal to a given Angle A. Fig. 8.

Draw any right Line ED to form a Triangle as EAD, take BF = AD, and upon BF make the Triangle BFG whose Side BG = AE, and GF = ED (by the last) then also the Angle B = A; if we suppose one Triangle be laid on the other, the Sides will mutually agree with each other, and therefore be equal, for if we consider these two Triangles are made of the same given three Lines, they are manifestly one and the same Triangle.

Otherwise

Upon

Plate II.

Upon the Centers A and B, at any Diſtance, let two Arcs, DE, FG, be deſcribed; make the Arc FG = DE, and thro' B and G draw the Line BG, and it is done.

For ſince the Chords ED, GF, are equal, the Angles A and B are alſo equal, as before (by Def. 19.)

PROB. III.

To biſect or divide into two equal Parts, any given right-lined Angle, BAC. Fig. 9.

In the Lines AB and AC, from the Point A ſet off equal Diſtances AE = AD, then, with any Diſtance, more than the half of DE, deſcribe two Arcs to cut each other in ſome Point F; and the right-line AF, joining the Points A, and F, will biſect the given Angle BAC.

For if DF and FE be drawn, the Triangles ADF, AEF, are equilateral to each other, *viz.* AD = AE, DF = FE, and AF common, wherefore DAF = EAF, as before.

PROB. IV.

To biſect a right line, AB. Fig. 10.

With any Diſtance, more than half the Line, from A and B, deſcribe two Circles CFD, CGD, cutting each other in the Points C and D; draw CD, interſecting AB in E, then AE = EB.

For

Plate II.

For if AC, AD, BC, BD, be drawn; the Triangles ACD, BCD, will be mutually equilateral, and consequently the Angle ACE = BCE: Therefore the Triangles ACE, BCE, having AC = BC, CE common, and the Angle ACE = BCE; (by Theo. 6.) the Base AE = the Base BE.

Cor. Hence it is manifest that CD not only bisects AB, but is perpendicular to it. (by Def. 11.)

PROB. V.

On a given Point A, in a right Line EF, to erect a Perpendicular. Fig. 11.

From the Point A lay off on each Side, the equal Distances, AC, AD; and from C and D, as Centers, with any Interval greater than AC or AD, describe two Arcs intersecting each other in B; from A to B draw the Line AB, and it will be the Perpendicular required.

For, let CB, and BD be drawn; then the Triangles CAB, DAB, will be mutually equilateral and equiangular, so CAB = DAB, a right Angle, (by Def. 11.)

PROB. VI.

To raise a Perpendicular on the End B of a right Line AB. Fig. 12.

From any Point D not in the Line AB, with the Distance from D to B, let a Circle be described cutting AB in E; draw from E thro' D the right Line EDC, cutting the Periphery in C, and join CB; and that is the Perpendicular required.

EBC

Plate II.

EBC being a Semicircle, the Angle EBC will be a right Angle, (by Cor. 5. Theo. 7)

PROB. VII.

From a given Point A, to let fall a Perpendicular upon a given right Line BC. Fig. 13.

From any Point D, in the given Line, take the Diſtance to the given Point A, and with it deſcribe a Circle AGE, make GE = AG, join the Points A and E, by the Line AFE, and AF will be the Perpendicular required.

Let DA, DE, be drawn; the Angle ADF = FDE, DA = DE, being Radii of the ſame Circle, and DF common; therefore (by Theo. 6.) the Angle DFA = DFE, and FA a Perpendicular. (by Def. 11.)

PROB. VIII.

Thro' a given Point A, to draw a right Line AB, parallel to a given right Line CD. Fig. 14.

From the Point A, to any Point, F, in the Line CD, draw the Line AF; with the Interval FA, and one Foot in F deſcribe the Arc AE, and with the like Interval and one Foot in A, deſcribe the Arc, BF, making BF = AE; thro' A and B draw the Line AB, and it will be parallel to CD.

By Prob. 2. The Angle BAF = AFE, and by Theo. 11. BA and CD are parallel.

Plate II.

PROB. IX.

Upon a given Line AB, to describe a Square ABCD. Plate I. Fig. 17.

Make BC perpendicular and equal to AB; and from A and C with the Line AB, or BC, let two Arcs be described cutting each other in D; from whence to A and C, let the Lines AD, DC be drawn; so is ABCD the Square required.

For all the Sides are equal by Construction; therefore the Triangles ADC, and BAC are mutually equilateral and equiangular, and ABCD is an equilateral Parallelogram, whose Angles are right. For B being right, D is also right, and DAC, DCA, BAC, ACB, each half a right (by Lemma preceding Theo. 7. and Cor. 2. Theo. 5.) whence DAB and BCD will each be a right Angle, and (by Def. 44.) ABCD is a Square.

SCHOLIUM.

By the same Method a Rectangle or Oblong may be described, the Sides thereof being given.

PROB. X.

To divide a given right Line AB, into any proposed Number of equal Parts. Fig. 15.

Draw the infinite right Line AP, making any Angle with AB, also draw BQ parallel to AP, in each of which let there be taken as many equal

Parts

Plate II.

Parts AM, MN, &c. Bo, on, &c. as you would have AB divided into; then draw Mm, Nn, &c. intersecting AB in E, F, &c. and it's done.

For MN, and mn, being equal and parallel, FN will be parallel to EN; and in the same Manner GO to FN (by Theo. 12.) therefore AM, MN, NO being all equal by Construction, it is plain (from Theo. 20.) that AE, EF, FG, &c. will likewise be equal.

PROB. XI.

To find a third Proportional to two given right Lines, A and B. Fig. 16.

Draw two infinite blank Lines CE, CD, any wise to make an Angle. Lay the Line A from C to F, and the Line B from C to G, and draw the Line FG; lay again the Line A from C to H, and thro' H, draw HI parallel to FG, (by Prob. 8.) so is CI the third Proportional required.

For by Cor. 1: Theo. 20. CG : CH : : CF : CI.

Or B : A : : A : CI.

PROB. XII.

Three right Lines A, B, C, given to find a fourth Proportional. (Fig. 17.)

Plate II.

Having made an Angle DEF any wise, by two infinite blank right Lines, ED, EF, as before; lay the Line A from E to G, the Line B from E to I, and draw the Line IG, lay the Line C from E to H, and (by Prob. 8.) draw HK parallel thereto, so will EK be the fourth Proportional required.

For by Cor. 1. Theo. 20. EG : EI : : EH : EK.

Or A : B : : C : EK.

PROB. XIII.

Plate III.

Two right Lines, A and B, given to find a Mean proportional. (*Fig.* 1.)

Draw an infinite blank Line, as AF, on which lay the Line A, from A to B, and the Line B, from B to C. on the Point B, which is the joining of the Lines A and B, erect a Perpendicular BD, (by Prob. 5.) bisect AC in E (by Prob. 4.) and describe the Semicircle ADC; and from the Point D, where its Periphery cuts the Perpendicular BD, draw the Line BD, and that will be the Mean Proportional required.

For if the Lines AD, DC, be drawn, the Angle ADC is a right Angle, (by Cor. 5. Theo. 7.) being an Angle in a Semicircle.

The Angles ABD, DBC, are right ones, (by Def. 11.) the Line BD being a Perpendicular; wherefore the Triangles ABD, DBC, are similar, thus the Angle ABD = DBC, being both right, the Angle DAC is the Complement of BDA to a right Angle, (by Cor. 2. Theo. 5.) and is therefore equal to BDC, the Angle ADC being a right Angle as before; consequently

Plate III.

ſequently (by Cor. 1. Theo. 5.) the Angle ADB = DCB, wherefore (by Theo. 16.)

AB : BD : : BD : BC.
Or A : BD : : BD : B.

PROB. XIV.

To divide a right Line AB, in the Point E, ſo that AE ſhall have the ſame Proportion to AB, as two given Lines C and D have. (*Fig.* 2.)

Draw an infinite blank Line, AF, to the Extremity of the Line AB, to make with it any Angle; lay the Line C from A to C, and D from A to D, and join the Points B and D, by the Line BD, thro' C draw CE parallel to BD (by Prob. 8.) ſo is E the Point of Diviſion.

For by Cor. 1. Theo. 20. AC : AD : : AE : AB.
Or C : D : : AE : AB.

PROB. XV.

To deſcribe a Circle about a Triangle ABC (or which is the ſame Thing) thro' any three Points, A, B, C, which are not ſituate in a right Line. (*Fig.* 3.)

By Prob. 4. Biſect the Line AC by the Perpendicular DE, and alſo CB, by the Perpendicular FG, the Point of Interſection H of theſe Perpendiculars is the Center of the Circle required, from which take the Diſtance to any of the three Points A, B, C, and deſcribe the Circle ABC and it is done.

For

Plate III.

For by Cor. to Theo. 8. The Lines DE and FG muſt each paſs thro' the Center, therefore their Point of Interſection H, muſt be the Center.

SCHOLIUM.

By this Method the Center of a Circle may be found, by having only a Segment of it given.

PROB. XVI.

To make an Angle of any Number of Degrees, at the Point A, of the Line AB, ſuppoſe of 45 *Degrees.* (*Fig.* 4.)

From a Scale of Chords take 60 Degrees, for 60 is equal to the Radius, (by Cor. Theo. 15.) and with that Diſtance from A as a Center, deſcribe a Circle from the Line AB; take 45 Degrees, the Quantity of the given Angle, from the ſame Scale of Chords, and lay it on that Circle from a to b, thro' A and b, draw the Line AbC; and the Angle A, will be an Angle of 45 Degrees as required.

If the given Angle were more than 90, take its half (or divide it into any two Parts leſs than 90) and lay them after each other on the Arc which is deſcribed with the Chord of 60 Degrees; thro' the Extremity of which and the Center, let a Line be drawn, and that will form the Angle required, with the given Line.

PROB.

Plate III.

PROB. XVII.

To meaſure a given Angle ABC. (Fig. 5.)

If the Lines which include the Angle be not as long as the Chord of 60 on your Scale, produce them to that or a greater Length, and between them ſo produced, with the Chord of 60 from B, deſcribe the Arc ed; which Diſtance ed meaſured on the ſame Line of Chords, gives the Quantity of the Angle BAC 48 Degrees as required; this is plain from Def. 19.

PROB. XVIII.

To make a Triangle BCE equal to a given Quadrilateral Figure ABCD. Fig. 6.

Draw the Diagonal AC, and Parallel to it (by Prob. 8.) DE, meeting AB produced in E; then draw CE, and ECB will be the Triangle required.

For the Triangles ADC, AEC being upon the ſame Baſe AC, and under the ſame Parallel ED, (by Cor. to Theo. 13.) will be equal, therefore if ABC be added to each, then ABCD = BEC.

PROB. XIX.

To make a Triangle DHF, equal to a given five-ſided Figure ABCDE. Fig. 7.

Draw DA and DB, and alſo EH and CF parallel to them, (by Prob. 8.) meeting AB produced in H and

Plate III.

H and F; then draw DH, DF, and the Triangle HDF is the one required.

For the Triangle DEA = DHA, and DBC = DFB (by Cor. to Theo. 13.) therefore by adding these Equations DEA + DBC = DHA + DFB, if to each of these ADB be added; then DEA + ADB + DBC = ABCDE = (DHA + ABD + DFB) = DHF.

PROB. XX.

To project the Lines of Chords, Sines, Tangents, and Secants to any Radius. Fig. 8.

On the Line AB, let a semicircle ADB be described; let CD be drawn perpendicular to the Center C, and the Tangent BE perpendicular to the End of the Diameter; let the Quadrants, AD, DB be each divided into 9 equal Parts, every of which will be 10 Degrees; if then from the Center C, Lines be drawn thro' 10, 20, 30, 40, *&c.* the Divisions of the Quadrant BD, and continued to BE, we shall there have the Tangents of 10, 20, 30, 40, *&c.* and the Secants C 10, C 20, C 30, *&c.* are transferred to the Line CD, produced by describing the Arcs 10, 10: 20, 20: 30, 30: *&c.* If from 10, 20, 30, *&c.* the Divisions of the Quadrant BD there be let fall Perpendiculars, let these be transferred to the Radius CD, and we shall have the Sines of 10, 20, 30, *&c.* and if from A we describe the Arcs 10, 10: 20, 20: 30: 30, *&c.* from every Division of the Arc AD; we shall have a Line of Chords. The same Way we may have the Sine Tangent, *&c.* to every single Degree on the

the Quadrant, by subdividing every of the 9 former Divisions into 10 equal Parts. By this Method the Sines, Tangents, &c. may be drawn to any Radius; and if after they be transferr'd to Lines a Rule, we shall have the Scales of Sines, Tangents, &c. ready for use.

Concerning Scales of equal Parts.

If an Inch be divided into any assigned Number of equal Parts, and if these Parts be continued on in a right Line, and if the last of them be subdivided into 10 equal Parts, and thence if the first Divisions be numbered with 1, 2, 3, 4, &c. as far as the Ruler upon which they are transferred will admit, the Scale is compleated.

These Numbers 1, 2, 3, 4, &c. usually stand for 10, 20, 30, 40, &c. and every one of the Subdivisions is called 1: But if the Numbers 1, 2, 3, 4, &c. be called 100, 200, 300, 400, &c. then every one of the Subdivisions will be 10, and the Units must be guessed at.

On one Side of most Surveying Scales, there are Lines, or Scales, marked at the End with 50, 45, 40, 35, 30, 25, 15, 10, and sometimes with other Numbers; these are Scales of so many Parts to an Inch, (whether of Feet, Yards, Perches, or Miles,) as the respective Number at the End of each expresses; but in the Surveying Way, they are counted to be so many Perches to an Inch, and sometimes so many Feet to an Inch.

On the contrary Side there are two Scales, one of 10, and the other of 20; or one of 100, and the

other of 200; or one of 1000, and the other of 2000 Parts to an Inch, diagonally divided; (a View of the Scale will make all easy:) The first of these *Surveyors* call a Scale of 10, and the other a Scale of 20 Perches to an Inch; and are thus counted: Every large Division is 10, every one of the Subdivisions is 1, and every one downwards is one tenth of a Perch; or sometimes thus, every large Division is called 100, every Subdivision 10, and every one downwards 1: Or again, frequently by *Navigators*, every large Division is called 1000, every Subdivision 100, every one downwards 10, and the tenth Part of the Distance between the Lines 1.

Hence it is easy to measure the Length of any Line, knowing the Scale by which it was laid down; and on the contrary, to set off any given Distance, from any Scale.

Of

Of LOGARITHMS.

IF to a Series of Numbers in *Geometrical Progression*, whose common *Ratio* is 10, and first Term 1; we annex another Series of Numbers in *Arithmetical Progression*, whose first Term is 0, and common Difference 1: These latter Numbers will be the Logarithms of the former.

Numbers.	Logarithms.
1	0.00000
10	1.00000
100	2.00000
1000	3.00000
10000	4.00000, &c.

If several Geometrical Means be taken, and the like Number of Arithmetical ones, to the corresponding Numbers, the latter will be the Logarithms of the former.

The Nature therefore of Logarithms is such, that Addition of them answers to the Multiplication of their corresponding Numbers; and Subtraction to Division: That is, when two Numbers proposed are to be multiplied into each other, if we take the Logarithms answering to those Numbers, and add them together, the Sum will be the Logarithm answering to the Number, which is the Product of the two proposed Numbers.

Again, when one Number is proposed to be divided by another; if from the Logarithm of the Dividend, we substract the Logarithm of the Divisor, the Remainder shall be the Logarithm of the Quotient.

Most Tables of Logarithms contain the Logarithms of all Numbers from 1 to 10000, the Column mark'd at the Top N, is that in which you must find your Number; in the same Line with which, in the adjacent Column, is the Logarithm of that Number.

EXAMPLE.

Required the Logarithm of 365.

Answer, 2.56229.

And though most Tables of Logarithms run but to 10000, yet by them the Log. of any Number not exceeding 10,000,000 may be found, and on the contrary, the Number to any such Logarithm, thus,

1. Find the Log. of the first four Figures of the given Number.

2. Take that Log. from the Log. of the Number next following, and note their Difference.

3. Multiply that Difference by the remaining Figures of the given Number; and from the Product, cut off as many Figures as remain in the given Number, or as the given Number is more than four (counting from the Right to the Left) as in Decimals.

4. The

4. The whole Number in the Product, added to the first Log. is the Log. required; but the first Figure, which is called the Index, or Characteristick, must be changed; and always be one less than the Number of Figures in the Logarithm.

Example I.

Required the Logarithm of the Number 3567894

The Log. of 3567, which are the first four Figures is	3.55230
The Log. of the following or next Number, *viz.* 3568 is	3.55242
Their Difference, - - - -	12
Mult. by the remaining Fig. *viz.*	.894
Cut off 3 Figures, because 894 is 3 Figures, and the Product is, - - - - - -	10.728
To which add the first Log.	3.55230
Their Sum is	3.55240

But because the given Number consists of 7 Figures, the Index must be one less which is 6; so the above Index 3, must be changed to 6, and we have 6.55240 the Log. of 3567894 required.

Example II.

Required the Log. of the Number 125607.

The Log. of 1256 is	3.09899
The next Log. following is	3.09934

Their

Their Difference is - - -	35
Multiply by .07 the remaining Figures	.07
Product - - - - - -	2.45
To which add the first Log.	3.09899
Their Sum is - - -	3.09901

Because the given Number consists of 6 Places, change the last Index to 5, which is one less than the Places in the given Number; and you have 5.09901, the Log. of 125607 required.

Because any Number consisting of both Integers and Decimals, is equal to the Quotient of the whole considered as an Integer, divided by the Denominator of the Decimal Part; and since by the Nature of Logarithms, Substraction in them answers the Quotient of other Numbers; therefore it follows, that when a Number is given consisting of Integers, and Decimals, we can find its Log. thus: Find the Log. of the whole considered as one Integer; then from that, take the Log of the Denominator of the Decimal Part; or (which is the same thing) from the Index of the Log. of the whole considered as an Integer, substract a Number less by one, than the Number of Places in the Denominator of the Fraction, and the Remainder will be the Log. required; or the Index of the Log. must be 1 less, than the Number of Figures in the Integer to which the Decimal is annex'd.

Example I.

What is the Log. of the Number 36.5?

Find the Log. of 365, which is 2.56229: then because 10 is the Denominator of the Decimal Part of the proposed Number, and 1.00000 its Log. therefore from 2.56229, take 1.00000, and there remains 1.56229 the Log. required. Or,

Or, becauſe the whole Number conſiſts of two Figures, the Index of the Log. muſt be one leſs, and is therefore 1.56229, as before.

Example II.

What are the Logs of 6543, 654.3, 65.43, 6.543, .6543, .06543 and .006543?

6543 - - - -	3.81578
654.3	2.81578
65.43	1.81578
6.543	0.81578
.6543	$\bar{9}.81578$
.06543	$\bar{8}.81578$
.006543	$\bar{7}.81578$

For the Log. of a Decimal Fraction is the ſame as that of an Integer; only the Index is Negative, and is ſo much leſs than 0. as the Place of the Decimal is removed from Unity; and thoſe Indices may be diſtinguiſhed from abſolute ones, by ſetting a Negative Sign over them, as above.

To find the Number of a given Logarithm.

Look for the given Log. amongſt the Logs from 1000 to 10000 (not regarding the Index or firſt Figure) and if you find the exact Log. you want, you have in the Margin the required Number. But if the Index of the given Log. be leſs than 3, cut off from the Number found, as many Figures as it is leſs; and the Figures ſo cut off will be Decimals, and the others Integers. Or if the firſt Figure, or Index, be greater than 3, add as many Cyphers to the Number found as it is more, and you have the Number required.

Exam-

EXAMPLES.

Find the Numbers correſpondent to the following Logarithms.

Given Logarithms.		Numbers.
5.55230	Anſwer	356700.
4.55230	—	35670.
3.55230	—	3567.
2.55230	—	356.7
1.55230	—	35.67
0.55230	—	3.567
9.55230	—	.3567
8.55230	—	.03567
7.55230	—	.003567, &c.

But if the exact Log. cannot be found in the Table, and the Number of Figures required, exceed four, then

1. Find as before (not regarding the Index) the Log. anſwering to the firſt four Figures, but leſs than the given Log.

2. Take that from the given one, and if the Remainder do not conſiſt of two Figures, prefix a Cypher to it; and after theſe two Figures annex three Cyphers, ſo will you have five Figures for a Dividend.

3. Divide that by the Difference between the Log. found, and the next following, and if your Quotient do not conſiſt of three Figures, prefix a Cypher or Cyphers to make it; which three Figures place after the firſt four found.

Then obſerve the Index of the given Log. which ſhews how many Figures muſt be Integers, and how many Decimals; for the Number of Integers

tegers is one more, than the given Index as before.

EXAMPLE I.

1. Required the Number of the Log. 4.55241

The nearest Log. which is less is 3.55230
its Number is 3567.

The Difference of these with three Cyphers is for a Dividend 11000

The Log. found 3.55230
The next Log. 3.55242

Their Difference will give for a Divisor 12

12)11000(916 Quotient
108

20
12

80

Which Quotient place after the first four Figures found, and you have 3567916; and because the Index is 4, the Number will be 35679.16 required.

2. Required the Number answering to the Log. 5.09901.

The nearest Log. to which is 3.09899 its No. 1256

Dividend 02000

Log. found 3.09899
Next Log. 3.09934

35 Divisor 35)02000(57
175

250
245

5

Because the Quotient consists of but two Figures, prefix a Cypher to it to make it three, and it is 057: which annex'd to the first four found, is 1256057; and because the Index of the given Log. is 5, its Number will be 125605.7.

From what has been said on this Head, the following Problems may easily be solv'd by Logarithms, *viz.*

PROB. I.

Multiply 134, by 25.6

To Log. of 134	2.12710
Add the Log. of 25.6	1.40824
Sum	3.53534

The Number answering to which Sum, *viz.* 3430, is nearly the Product of 134 by 25.6 and is the Answer.

Again, Multiply 234 by 36.

To

To Log. of	234	2.36922
Add the Log. of	36	1.55630
	Sum	3.92552 its Number

is 8424 required.

PROB. II.

What's the Quotient of 828 by 23?

From the Log. of	828	2.91803
Take the Log. of	23	1.36173
	Difference	1.55630 its Number

36, the Quotient required.

Again, what's the Quotient of 30550 by 47?

From the Log. of	30550	4.48501
Take the Log. of	47	1.67210
		2.81291 its Numb.

is 650 the Quotient required.

PROB. III.

Three Numbers in a direct Proportion given, to find a Fourth.

From the Sum of the Logarithms of the second and third Numbers; deduct the Logarithm of the first, the Remainder will be the Logarithm of the fourth required:

EXAMPLE I.

Let the three proposed Numbers be 36, 48, 66, to find a fourth proportional.

To Log. of 48	1.68124
Add Log. of 66	1.81954
Product	3.50078
Take Log. of 36	1.55630
	1.94448

the Number is 88, the fourth required.

Again, let three Numbers be 240, 1440, 1230' to find a fourth proportional.

To the Log. of 1440	3.15836
Add the Log. of 1230	3.08991
From the Product	6.24827
Take the Log. of 240	2.38021
	3.86806

its Number 7380 the 4th required.

PROB. IV.

To find the Square of any given Number.

Multiply the given Numbers Logarithm by 2, and the Product is the Logarithm of its Square.

EXAM-

EXAMPLE.

Required the Square of 36.

Log. of 36	1.55630
Multiply by	2
	3.11260

its Number 1296 the Square required.

PROB. V.

To extract the Square Root of any given Number.

Take half of the Logarithm of the Number, and that is the Logarithm of its Square Root.

EXAMPLE.

Required the Square Root of 1296.

Log. of 1296	3.11261
Its half is	1.55630 its Number is 36,

the Square Root of the Number required.

By the Manner of Projecting the Lines of Chords, Sines, Tangents and Secants (being Prob. 20 of Geometry) it is evident, that if the Radius be supposed any Number of equal Parts (as 1000 or 10000, *&c.*) the Sine, Tangent, *&c.* of every Arc, must consist of some Number of those equal Parts; and by computing them in Parts of the Radius,

dius, we have Tables of Sines, Tangents, &c. to every Arc of the Quadrant called natural Sines, Tangents, &c. and the Logarithms of these, give us Tables of logarithmic Sines, Tangents, &c. and such are usually bound up with Logarithms of Numbers.

In which you may observe, that each Page is divided into 8 Columns, the first and last of which are Minutes, and the intermediate ones contain the Sines, Tangents and Secants, the upper and lower Columns contain Degrees, the Column of the Minutes on the left Hand of each Page, answers to the Degrees in the Top Column; and the Sines, Tangents and Secants belonging to those Degrees and Minutes, are in the Columns mark'd at the Top with the Words Sine, Tangent, and Secant; the Column of Minutes on the right Hand of each Page answers to the Degrees in the Bottom of the Page; and the Sines, Tangents and Secants, answering to those Degrees and Minutes, are in the Columns, mark'd at the Bottom with the Word Sine, Tangent, Secant; the Degrees in the Top Column beginning at 0, proceed to 44 where they end; and those at the Bottom of the Page begin at 89, and proceed to 45 in a decreasing Series; the Degrees in the different Columns being the Complement of each other. From what has been said, we may easily find the Sine, Tangent or Secant of any Arc from the Tables, by looking for the given Number of Degrees, at the Head or Foot of the Page, according as they are less or greater than 45, and in the proper Side Column for the odd Minutes, if there be any; then below or above the Word Sine, Tangent or Secant, and on the same Line with the Minutes, we shall have that which was required.

EXAM-

EXAMPLE I.

Required the Sine of 36 Degrees 40 Minutes.

Look at the Head of the Page for 36 Degrees, and in the Side Column on the left Hand, for 40 Minutes; then below the Word Sine, on the ſame Line with 40, we find 9.77609; which is that required.

EXAMPLE II.

Required the Tangent of 54 Degrees 30 Minutes.

Look at the Foot of the Page, (becauſe the propoſed Degrees are more than 45.) for 54 Degrees, and in the right Hand Column for 30 Minutes; then in the Column mark'd Tangent at its Bottom, and on the ſame Line with the 30 Minutes, in the Side Column, we find 10,14673, which is the Log-tangent required.

The Reverſe of this, *viz.* The Logarithm of a Sine, Tangent, or Secant, being given, to find the Arc belonging to it, is performed by only looking in the proper Column for the neareſt Logarithm to that propoſed, and the Degrees and Minutes anſwering thereto, are thoſe required.

We will now ſhew how any Sine, Tangent, or Secant may be had, tho' the Figures in the Tables were defaced, miſprinted, or obliterated.

PROB. I.

To find the Tangent which is defaced, by the Sine and Co-ſine.

The

The Co-sine taken from the Sine added to 90, or Radius, which is 10.00000, the Remainder is the Tangent. (by Part 1. Theo. 24.)

EXAMPLE.

1. Suppose the Tangent of 41°. 20ʹ, was defaced, but the Sine and Co-sine of it visible.

From the Sine of 41°. 20ʹ + 10.00000, or Radius, — — — —	19.81983
Take the Co-sine of 41°. 20ʹ	9.87557
The Rem. is the Tan. of 41°. 20ʹ req. *viz.*	9.94426

2. To find a Sine which is misprinted, by Help of the Co-sine and Tangent.

From the Sum of the Tangent and Co-sine, take 10.00000, or Radius, or (which is the same Thing) cut off the first Figure of the Index, the Remainder is the Sine required, (by Part 2. Theo. 24.)

EXAMPLE.

Suppose the Sine of 46°. 50ʹ was defaced, but the Tangent and Co-sine visible.

To the Tangent of 46°. 50ʹ	10.02781
Add the Co-sine of 46°. 50ʹ	9.83513
Their Sum is the Sine of 46°. 50ʹ req. *viz.*	9.86294

The Co-tangent and Co-sine of any Arc may be had by the same Method; the Complement of any Degree, being only its Residue from 90, or a Quadrant

Quadrant, as before observed, (by Theo. 24. Part 3 and 4.)

3. To find a Tangent by the Help of a Co-tangent only.

From twice the Radius, which is 20.00000; take the Co-tangent, the Remainder is the Tangent, (by Theo. 24. Part 5.)

EXAMPLE.

Required the Tangent of 29°. 50' being defaced, as also the Sine and Co-sine defaced, by the Co-tangent only.

From twice the Radius,	20 00000
Take the Co-tangent of 29°. 50'	10.24148
The Rem. is the Tang. of 29°. 50' req.	9 75852

4. To find the Secant, by the Help of a Co-sine; which may be found of great Use, when a Table of Sines and Tangents can only be had.

From twice the Radius, which is 20 00000 take the Co-sine, and the Remainder will be the Secant, (by Theo. 24. Part 6)

EXAMPLE.

Required the Secant of 57°. 20' by the Help of the Co-sine only.

From the double Radius,	20.00000
Take the Co-sine of 57°. 20'	9.73219
The Rem. is the Secant of 57°. 20' req.	10.26781

5. To find a Secant by the Help of the Sine and Tangent.

From the Tangent added to Radius, take the Sine, the Remainder will be the Secant, (by Theo. 24. Part 7.)

EXAMPLE.

Required the Secant of 57°. 20' by Help of the Sine and Tangent.

From the Tan. of 57°. 20' + 10.00000 the Radius,	20.19303
Take the Sine of 57°. 20'	9.92522
The Rem. is the Secant of 57°. 20' req.	10.26781

The Secants in these Tables might have been omitted, because all Proportions in which they are concerned may be wrought by Sines and Tangents only, as shall be shewn in the several Cases of Plane Trigonometry; and are here only inserted that all the various Methods of resolving Triangles may be shewn.

SECT.

SECT. II.

Containing Plane Trigonometry, right-angled and oblique; with its Application in determining the Measures of inacceſſible Heights and Diſtances.

Plane Trigonometry.

IS the Science of meaſuring the Sides and Angles of Plane Triangles. It is divided into two Parts, *viz.* into *Rectangular* and *Oblique angular Trigonometry*, becauſe every Triangle is either right-angled or oblique; and therefore we ſhall begin with

RECTANGULAR TRIGONOMETRY.

Plate V. Fig. 1.

1. In every right-angled Plane Triangle ABC, if the Hypothenuſe AC be made the Radius, and with it a Circle, or an Arc of one, be deſcribed from each End; it is plain (from Def. 22.) that BC is the Sine of the Angle A, and AB is the Sine of the Angle C; that is, the Legs are the Sines of their oppoſite Angles.

Plate V.

2. If one Leg AB be made the Radius, and with it, on the Point A, an Arc be described; then BC is the Tangent, and AC is the Secant of the Angle A, by Def. 24 and 25. Fig. 2.

3. If BC be made the Radius, and an Arc be described with it on the Point C; then is AB the Tangent, and AC is the Secant of the Angle C, as before. Fig. 3.

Because the Sine, Tangent, or Secant of any given Arc in one Circle, is to the Sine, Tangent, or Secant of a like Arc (or to one of the like Number of Degrees,) in another Circle; as the Radius of the one is to the Radius of the other; therefore the Sine, Tangent, or Secant of any Arc is proportional to the Sine, Tangent, or Secant of a like Arc, as the Radius of the given Arc is to 10.00000, the Radius from whence the Logarithmic Sines, Tangents and Secants, in most Tables, are calculated, i. e.

If AC be made the Radius, the Sines of the Angle A and C, described by the Radius AC, will be proportional to the Sines of the like Arcs, or Angles in the Circle, that the Tables now mentioned were calculated for. So if BC was required, having the Angles and AB given, it will be Fig 1.

As S.C : AB : : S.A : BC.

i. e. As the Sine of the Angle C in the Tables, is to the Length of AB; (or Sine of the Angle C, in a Circle whose Radius is AC); so is the Sine of the

Plate V.

the Angle A in the Tables, to the Length of BC. (or Sine of the ſame Angle, in the Circle, whoſe Radius is AC.)

In like Manner, the Tangents and Secants, repreſented by making either Leg the Radius; will be proportional to the Tangents and Secants of a like Arc, as the Radius of the given Arc is to 10.00000, the Radius of the Tables aforeſaid.

Hence it is plain, that if the Name of each Side of the Triangle be placed thereon, a Proportion will ariſe to anſwer the ſame End as before: Thus if AC be made the Radius, let the Word Radius be wrote thereon; and as BC, and AB, are the Sines of their oppoſite Angles; upon the firſt let S.A, or Sine of the Angle A, and on the other, let S. C, or Sine of the Angle C, be wrote: Then if a Side be required, it may be obtained by this Proportion, *viz.*

As the Word on the Side given,
  Is to the Side given,
So is the Word on the Side required
  to the Side required.

Thus if the Angles A and C, and the Hypothenuſe AC were given, to find the Legs; the Proportions will be

1. R : AC : : S. A : BC. Fig. 1.

That is, as Radius is to AC; ſo is the Sine of the Angle A, to BC. And,

2. R : AC : : S. C : AB.

That is, as Radius is to AC; ſo is the Sine of the Angle C, to AB.

But

Plate V.

But when an Angle is required, we uſe this Proportion, *viz.*

As the Side that is made the Radius,
is to Radius;
So is the other given Side,
to the Word upon it.

Thus, if the Legs were given, to find the Angle A, and if AB be made the Radius, it will be,

AB : R : : BC : T. A. Fig. 2.

That is, as AB, is to Radius; ſo is BC, to the Tangent of the Angle A.

After the ſame Manner, the Sides or Angles of all right-angled plane Triangles may be found, from their proper Data.

We here, in Plate 4, give all the Proportions requiſite for the Solution of the ſix Caſes in Rectangular Trigonometry; making every Side poſſible the Radius.

In the following Triangles, this Mark — in an Angle, denotes it to be known, or the Quantity of Degrees it contains to be given; and this Mark ' on a Side, denotes its Length to be given, in Feet, Yards, Perches, or Miles, *&c.* and this Mark °, either in an Angle, or on a Side, denotes the Angle or Side required.

From

Plate V.

From theſe Proportions it may be obſerved; that, to find a Side, when the Angles and one Side are given, any Side may be made the Radius: And to find an Angle, one of the given Sides muſt be made the Radius. So that in the 1ſt, 2d, and 3d Caſes, any Side, as well required as given, may be made the Radius; and in the firſt Statings of the 4th, 5th and 6th Caſes, a given Side only is made the Radius.

RECTANGULAR TRIGONOMETRY.

CASE I.

THE *Angles and Hypothenuſe given, to find the Baſe and Perpendicular. Fig.* 4.

In the right-angled Triangle ABC, ſuppoſe the Angle A $46^{\circ}. 30^{I}$. and conſequently the Angle C $43^{\circ}. 30^{I}$. (by Cor. 2. Theo. 5); and AC 250 Parts, (as Feet, Yards, Miles, *&c.*) required the Legs AB, and BC.

Geometrically.

Make an Angle of $46^{\circ}. 30^{I}$, in blank Lines, (by Prob. 16. Sect. 1.) as CAB; lay 250, which is the given Hypothenuſe, from a Scale of equal Parts, from A to C; from C, let fall the Perpendicular BC, (by Prob. 7. Sect. 1.) and that will conſtitute the Triangle ABC. Meaſure the Lines BC, and AB, from the ſame Scale of equal Parts, that AC was taken from; and you have the Anſwer.

By

Plate V.

By Calculation.

1. *Making AC the Radius*, the required Sides are found by these Proportions, as in Plate 4, Case 1.

R : AC : : SA : B.C.
R : AC : : SC : A.B.

i. e	As Radius	90°	10.00000
	is to AC,	250	2.39794
	So is the Sine of A	46°. 30′	9.86056
	to BC,	181. 4	2.25850

As Radius,	90°	10.00000
is to AC,	250	2.39794
So is the Sine of C	43°. 30′	9.83781
to AB,	172. 1	2.23575

If from the Sum of the second and third Logs, that of the first be taken, the Remainder will be the Log. of the fourth; the Number answering to which will be the Thing required; but when the first Log. is Radius, or 10.00000, reject the first Figure of the Sum of the other two Logs; (which is the same Thing as to subtract 10.00000); and that will be the Log. of the Thing required.

2. *Making AB the Radius.*

Secant A : AC : : R : AB.
Secant A : AC : : T. A : BC.

i. e.

i. e.	As the Secant of A	46°. 30′	10.16219
	is to AC	250	2.39794
	So is Radius	90°	10.00000
			12.39794
	to AB	172. 1	2.23575

As the Secant of A	46°. 30′	10 16219
is to AC	250	2.39794
So is the Tangent of A	46°. 30′	10.02275
		12.42069
to BC	181 . 4	2.25850

3. *Making BC the Radius.*.

Sec. C : AC : : R : BC.

Sec. C : AC : : T . C : AB.

i. e.	As the Secant of C	43°. 30′	10.13944
	is to AC	250	2.39794
	So is Radius	90°.	10.00000
			12.39794
	to BC	181 . 4	2.25850

As the Secant of C	43°. 30′	10.13944
is to AC	250	2.39794
So is the Tangent of C	43°. 30′	9.97725
		12.37519
to AB	172 . 1	2.23575

Or having found one Leg, the other may be obtained by Cor. 2. Theo. 14. Sect. 1.

By Gunter's *Scale.*

On this Scale there are Lines of Numbers, Sines,

and Tan. as well as Lines of Sine and Tan. Rumbs, Versed Sines, Meridional Parts, and Equal Parts But the three first Lines are sufficient for our present Purpose.

The Divisions on these respective Lines, are the Logarithms of Numbers, Sines, and Tangents, taken from a Scale of equal Parts, and applied on the Lines of the Scale.

The first and third Terms in the foregoing Proportions being of a like Nature, and those of the second and fourth being also like to each other; and the Proportions being direct ones, it follows; that if the third Term be greater, or less than the first, the fourth Term will be also greater or less than the second: Therefore the Extent in your Compasses from the first to the third Term, will reach from the second to the fourth.

Thus, to extend the first of the foregoing Proportions;

1. Extend from 90° to 46° . 30′, on the Line of Sines; that Distance will reach from 250 on the Line of Numbers, to 181, for BC.

2. Extend from 90° to 43° . 30′ on the Line of Sines; that Distance will reach from 250 on the Line of Numbers, to 172, for AB.

If the first Extent be from a greater, to a less Number; when you apply one Point of the Compasses to the second Term, the other must be turned to a less; and the contrary.

By Def. 22. Sect. 1. The Sine of 90° is equal to the Radius; and the Tangent of 45° is also equal to the Radius; because if one Angle of a right-angled

angled Triangle be 45°, the other will be alſo 45°; and thence (by the Lemma preceding Theo. 7. Sect. 1.) the Tangent of 45° is equal to the Radius: For this Reaſon the Line of Numbers of 10.00000, the Sine of 90°, and Tangent of 45°, being all equal, terminate at the ſame End of the Scale; where there are ſmall Braſs Centers, uſually placed to preſerve the Scale.

It was ſaid before, that the Tangents ended at 45°; but becauſe the Logarithms of Tangents more than 45°, muſt paſs off the Scale; ſuch Diſtances therefore as exceed 45°, are ſet backwards from 45, and numbered 50, 60, 70, &c.

There is no Line of Secants on the Scale; for every Thing requiſite, can be performed without them.

Thus the two firſt Statings of this Caſe anſwer the Queſtion without a Secant: The like will be alſo made evident in all the following Caſes.

C A S E 2.

The Baſe and Angle given; to find the Perpendicular and Hypothenuſe.

Plate V. Fig. 5.

In the Triangle ABC, there is the Angle A 42°. 20', and of Courſe the Angle C 47°. 40', (by Cor. 2. Theo. 5) and the Leg. AB 190, given; to find BC and AC.

Geometrically.

Make the Angle CAB (by Prob. 16. Sect. 1,) in blank Lines as before. From a Scale of equal Parts lay 190 from A to B; on the Point B erect a Perpendicular BC (by Prob. 5. Sect. 1.) the Point

where this cuts the other blank Line of the Angle, will be C; ſo is the Triangle ABC conſtructed: Let AC, and BC be meaſured, from the ſame Scale of equal Parts, that AB was taken from, and you have the Anſwer.

By Calculation.

1. *Making AC the Radius,*

S. C : AB : : R : AC.
S. C : AB : : S. A. BC.

i. e.	As the Sine of C	47°. 40′	9.86879
	is to AB,	190	2.27875
	So is Radius,	90°	10.00000
			12.27875
	to AC,	257	2.40996

As the Sine of C	47°. 40′	9.86879
is to AB	190	2.27875
So is the Sine of A	42°. 20′	9.82830
		12.10705
to BC,	173. 1	2.23826

2. *Making AB the Radius.*

R : AB : : T. A : B.C.
R : AB : : Sec. A : A.C.

i. e	As Radius	90°	10.00000
	is to AB,	190	2.27875
	So is the Tangent of A	42°. 20′	9.95952
	to BC	173. 1	2.23827

As

As Radius,	90°	10.00000
is to AB,	190	2.27875
So is the Secant of A	42°. 20′	10.13121
to AC	257	2.40996

3. *Making BC the Radius.*

T. C : AB : : Sec. C : AC.

T. C : AB : : R : BC.

i. e. As the Tangent of C	47°. 40′	10.04048
is to AB,	190	2.27875
So is the Secant of C	47°. 40′	10.17170
		12.45045
to AC,	257	2.40997

As the Tangent of C	47°. 40′	10.04048
is to AB,	190	2.27875
So is Radius,	90°	10.00000
		12.27875
to BC,	173. 1	2.23827

Or having found one of the required Sides, the other may be obtained, by one or the other of the Cors. to Theo. 14. Sect. 1.

By Gunter's Scale.

1. When AC is made the Radius.

Extend from 47° 40′, to 90°, on the Line of Sines;

Sines; that Diſtance will reach from 190 to 257, on the Line of Numbers, for AC.

2. When AB is made the Radius, the firſt Stating is thus performed.

Extend from 45° on the Tangents, (for the Tangent of 45° is equal to the Radius, or to the Sine of 90°, as before) to 42°. 20′; that Extent will reach from 190, on the Line of Numbers, to 173, for BC.

3. When BC is made the Radius, the ſecond Stating is thus performed.

Extend from 47°. 40′, on the Line of Tangents, to 45°, or Radius; that Extent will reach from 190 to 173, on the Line of Numbers, for BC; for the Tangent of 47°. 40′, is more than the Radius; therefore the fourth Number muſt be leſs than the ſecond, as before.

The two firſt Statings of this Caſe, anſwer the Queſtion without a Secant.

CASE 3.

The Angles and Perpendicular given; to find the Baſe and Hypothenuſe.

Plate V. Fig. 6.

In the Triangle ABC, there is the Angle A 40°, and conſequently the Angle C 50°, with BC 172, given; to find AC and AB.

Geo-

Geometrically.

Make an Angle CAB of 40° in blank Lines; (by Prob. 16. Sect. 1.) with BC 172, from a Line of equal Parts, draw the popped Lines EF parallel to AB (by Prob. 8. Sect. 1.) the lower Line of the Angle, and from the Point where it cuts the other Line in C, let fall a Perpendicular BC (by Prob. 7. Sect. 1.) and the Triangle is constructed: The Measures of AC, and AB, from the same Scale that BC was taken, will answer the Question.

What has been said in the two foregoing Cases, is sufficient to render the Operations in this, both by *Calculation* and *Gunter's Scale*, so obvious, that it is needless to insert them; however, for the sake of the Learner, we give for

Answer, AC 264. 5, and AB 202. 6.

CASE 4.

The Base and Hypothenuse given; to find the Angles and Perpendicular.

Plate V. Fig. 7.

In the Triangle ABC, there is given, AB 300, and AC 500: The Angles A and C, and the Perpendicular BC, are required.

Geometrically.

From a Scale of equal Parts, lay 300 from A, to B; on B erect an infinite blank perpendicular Line, with AC 500, from the same Scale, and one Foot of the Compass in A, cross the perpendicular Line

Line in C; and the Triangle is conſtructed.

By Prob. 17. Sect. 1. Meaſure the Angle A, and let BC be meaſured from the ſame Scale of equal Parts that AC and AB were taken from; and you have the Anſwer.

By Calculation.

1. *Making AC the Radius.*

AC : R : : AB : S. C.
R : AC : : S. A : BC.

i. e: As AC	500	2.69897
is to Radius,	90°	10.00000
So is AB	300	2.47712
		12.47712
to the Sine of C	36°. 52'	9.77815

By Cor. 2. Theo. 5. $90° - 36°. 52' = 53°. 08'$ the Angle A.

As Radius,	90°	10.00000
is to AC,	500	2.69897
So is the Sine of A	53°. 08'	9.90301
to BC,	400	2.60198

2. *Making AB the Radius.*

AB : R : : AC : Sec. A.
R : AB : : T. A : BC.

i. e.

i. e. As AB,	300	2.47712
is to Radius,	90°	10.00000
So is AC,	500	2.69897
		12.69897
to the Secant of A,	53°. 08'	10.22185

As Radius,	90°	10.00000
is to AB,	300	2.47712
So is the Tangent of A,	53°. 08'	10.12499
to BC,	400	2.60211

Or BC may be found from Cor. 1. Theo. 14. Sect. 1.

By Gunter's Scale.

1. Making AC the Radius.

Extend from 500 to 300, on the Line of Numbers; that Extent will reach from 90°. on the Line of Sines to 36°. 52', for the Angle C.

Again. Extend from 90° to 53°. 08', on the Line of Sines, that Extent will reach from 500 to 400, on the Line of Numbers, for BC.

2. Making AC the Radius the second Stating is thus performed.

Extend from Radius, or the Tangent of 45°, to 53°. 08', that Extent will reach from 300 to 400, for BC.

CASE V.

The Perpendicular and Hypothenuſe given, to find the Angles and Baſe.

Plate V. Fig. 8.

In the Triangle ABC, there is BC 306, and AC 370, given; to find the Angles A and C, and the Baſe AB.

Geometrically.

Draw a blank Line from any Point, in which at B, erect a Perpendicular, on which lay BC 306, from a Scale of equal Parts: From the ſame Scale, with AC 370, in the Compaſſes, croſs the firſt drawn blank Line in A, and you have the Triangle ABC conſtructed.

Meaſure the Angle A (by Prob. 17. Sect. 1.); and alſo AB, from the ſame Scale of equal Parts the other Sides were taken from, and you have the Anſwer.

The Operations by Calculation, the Square Root, and *Gunter's* Scale are here omitted, as they have been heretofore fully explained: The Statings, or Proportions muſt alſo be obvious, from what has already been ſaid.

Anſwer. The Angle A 55°. 48'; therefore the Angle C 34°. 12', and AB 208.

CASE 6.

The Base and Perpendicular given; to find the Angles and Hypothenuse.

Plate V. Fig. 9.

In the Triangle ABC, there is AB 225, and BC 272, given; to find the Angles A and C, and the Hypothenuse AC.

Geometrically.

Draw a blank Line, on which lay AB 225, from a Scale of equal Parts; at B erect a Perpendicular, on which lay BC 272, from the same Scale; join A and C, and the Triangle is constructed.

As before, let the Angle A, and the Hypothenuse AC be measured; and you have the Answer.

By Calculation.

1. *Making AB the Radius.*

AB : R : : BC : T. A.
R : AB : : Sec. A : AC.

2. *Making BC the Radius.*

BC : R : : AB : T. C.
R. : BC : : Sec. C : AC.

By Calculation; the Answer from the foregoing Proportions is easily obtained, as before.

But becauſe AC, by either of the ſaid Proportions, is found by Means of a Secant; and ſince there is no Line of Secants on *Gunter's* Scale; after having found the Angles, as before, let us ſuppoſe AC the Radius, and then

1. S. A : BC : : R. : AC.
or 2. S. C : AB : : R. : AC.

Theſe Proportions may be eaſily reſolved, either by *Calculation*, or *Gunter's Scale*, as before; and thus the Hypothenuſe AC may be found without a Secant.

From the two given Legs, the Hypothenuſe may be eaſily obtained, from Cor. 1. Theo. 14. Sect. 1.

Thus to the Square of AB=50625
Add the Square of BC= 73984

```
          124609(353=AC
          9
         ----
       65)346
          325
         -----
      703)2109
          2109
```

From what has been ſaid on Logarithms, it is plain,

1. That half the Logarithm of the Sum of the Squares of the two Sides, will be the Logarithm of the Hypothenuſe. Thus

The Sum of Squares, as before, is 124609; its Log. is 5.09554, the half of which is 2.54777; and

and the correſponding Number to this, in the Tables, will be 353, for AC.

2. And that half of the Logarithm of the Difference of the Squares of AC and AB, or of AC and BC, will be the Logarithm of BC, or of AB.

The following Examples are inſerted for the Uſe of the Learner.

1. Given,	the Angle C	$64^{\circ}.\ 40'$	AB	required.
	AC	3876	BC	
2. Given,	the Angle C	$47^{\circ}.\ 20'$	AC	required.
	AB	17	BC	
3. Given,	the Angle C	$28^{\circ}.\ 30'$	AB	required.
	BC	27187	AC	
4. Given,	AB	2	the Angles	required.
	AC	3	and BC	
5. Given,	BC	17	the Angles	required.
	AC	21.6	and AB	
6. Given,	AB	2871.64	the Angles	required.
	BC	3176.2	and AC	

The Anſwers are omitted, that the Learner may reſolve them himſelf by the foregoing Methods; by which Means he will find and ſee more diſtinctly their mutual Agreements; and become more expert, and the better acquainted with the Subject.

OBLIQUE

OBLIQUE ANGULAR PLANE TRIGONOMETRY.

BEFORE we proceed to the Solution of the four Cases of Oblique-Angular Triangles, it is necessary to premise the following Theorems.

THEO. I.

Plate V.

In any plane Triangle ABC, the Sides are proportional to the Sines of their opposite Angles, i. e. *S. C: AB : : S. A : BC, and S. C : AB : : S. B : AC; also S. B : AC : : S. A : BC.* Fig. 10.

By Theo. 10. Sect. 1. the half of each Side is the Sine of its opposite Angle; but the Sines of those Angles, in Tabular Parts, are proportional to the Sines of the same in any other Measure; and therefore the Sines of the Angles will be as the Halves of their opposite Sides: And since the Halves are as the Wholes, it follows, that the Sines of their Angles are as their opposite Sides, i. e. S. C : AB : : S. A : BC, *&c.* Q. E. D.

THEO. II.

In any plane Triangle ABC, the Sum of the two given Sides AB and BC, including a given Angle ABC, is to their Difference; as the Tangent of half the Sum of the two unknown Angles A and C is to the Tangent of half their Difference. Fig. 11.

Produce

Plate V.

Produce AB, and make HB=BC, and join HC: Let fall the Perpendicular BE, and that will bisect the Angle HBC (by Theo. 9. Sect. 1.) through B draw BD parallel to AC, and make HF = DC, and join BF; take BI = BA, and draw IG parallel to BD, or AC.

It is then plain, that AH will be the Sum, and HI the Difference of the Sides AB, and BC: And since HB = BC, and BE perpendicular to HC, therefore HE = EC (by Theo. 8. Sect. 1.); and since BA=BI, and BD and IG parallel to AC, therefore GD = DC = FH, and consequently HG = FD, and $\frac{1}{2}$ HG = $\frac{1}{2}$ FD, or ED. Again, EBC, being half HBC, will be also half the Sum of the Angles A and C, by Theo. 4. Sect. 1. Also since HB, HF, and the included Angle H, are severally equal to BC, CD, and the included Angle BCD; therefore (by Theo. 6. Sect. 1.) HBF = DBC = BCA (by Part 2. Theo. 3. Sect. 1.) and since HBD = A (by Part. 3. Theo. 3. Sect. 1.) and HBF = BCA; therefore FBD is the Difference, and EBD, half the Difference of the Angles A and C: Then making BE the Radius, it is plain that EC will be the Tan. of half the Sum, and ED the Tangent of half the Difference of the two unknown Angles A and C: Now IG being parallel to AC; AH : IH : : CH : GH. (by Cor. 1. Theo. 20. Sect. 1.) But the Wholes are as their Halves, i. e. AH : IH : : CE : ED, that is, as the Sum of the two Sides AB and BC, is to their Difference; so is the Tangent of half the Sum of the two unknown Angles A and C, to the Tangent of half their Difference. Q. E. D.

Plate V.

THEO. III.

In any right-lined plane Triangle ABD; the Base AD, will be to the Sum of the other Side, AB, BD, as the Difference of those Sides, is to the Difference of the Segments of the Base, made by the Perpendicular BE; viz. *the Difference between AE and ED. Fig.* 12.

Produce BD, till BG = AB the leſſer Leg; and on B as a Center, with the Diſtance BG or BA deſcribe a Circle AGHF; which will cut BD, and AD, in the Points H, and F: Then it is plain, that GD, will be the Sum, and HD, the Difference of the Sides AB and BD; alſo ſince AE = EF, (by Theo. 8. Sect. 1.) therefore FD, is the Difference of AE, and ED, the Segments of the Baſe: But (by Theo. 17. Sect. 1.) AD : GD : : HD : FD; that is the Baſe is to the Sum of the other Sides, as the Difference of thoſe Sides, is to the Difference of the Segments of the Baſe. Q. E. D.

THEO. IV.

If to half the Sum of two Quantities, be added half their Difference; the Sum will be the greateſt of them: and if from half the Sum be ſubtracted half their Difference, the Remainder will be the leaſt of them. Fig. 13.

Let the two Quantities be repreſented by AB, and BC; (making one continued Line;) whereof AB is the greateſt, and BC the leaſt; biſect the whole Line AC in E; and make AD = BC; then

its

Plate V.

it is plain, that AC is the Sum, and BD, the Difference of the two Quantities; and AE, or EC, their half Sum, and DE or EB their half Difference. Now if to AE we add EB, we ſhall have AB the greateſt Quantity; and if from EC we take EB, we ſhall have BC, the leaſt Quantity. Q. E. D.

Cor. Hence, if from the greateſt of two Quantity, we take half the Difference of them, the Remainder will be half their Sum; or if to half their Difference be added the leaſt Quantity, their Sum will be half the Sum of the two Quantities.

OBLIQUE ANGULAR TRIGONOMETRY.

TWO Sides, and an Angle oppoſite to one of them given, to find the other Angles and Side.

In the Triangle ABC, there is given AB 240, *the Angle A* 46°. 30', *and BC* 200; *to find the Angle C being acute, the Angle B, and the Side AC. Fig.* 14.

Geometrically.

Draw a blank Line, on which ſet AB 240 from a Scale of equal Parts; at the Point A, of the Line AB, make an Angle of 46°. 30', by an infinite blank Line; with BC 200, from a like Scale of equal Parts that AB was taken, and one Foot in B deſcribe the Arc DC to cut the laſt blank Line in the Points D and C. Now if the Angle C had been required obtuſe; Lines from D to B, and

Plate V.

to A, would conſtitute the Triangle; but as it is required acute, draw the Lines from C to B, and to A; and the Triangle ABC is conſtructed. From a Line of Chords let the Angles B and C be meaſured; and AC from the ſame Scale of equal Parts, that AB and BC were taken; and you will have the Anſwer required.

By Calculation.

This is performed by Theo. 1. of this Sect. thus;

As BC	200	2.30103
is to the Sine of A	46° 30′	9.86056
So is AB	240	2.38021
		12.24077
To the Sine of C	60°. 31′	9.93974

180 — the Sum of the Angles A and C, will give the Angle B; by Cor. 1. Theo. 5. Sect. 1.

A 46°. 30′
C 60. 31

180 — 107. 10 = 72° 59′ = B.

As the Sine of A	46°. 30′	9.86056
is to BC	200	2.30103
So is the Sine of B	72°. 59′	9.98056
		12.28159
To AC	263. 7	2.42103

By

Plate V.

By Gunter's Scale.

Extend from 200 to 240, on the Line of Numbers; that Distance will reach from 46°. 30' on the Line of Sines, to 60°. 31' for the Angle C.

Extend from 46°. 30', to 72°. 59', on the Line of Sines; that Distance will reach from 200 to 263.7, on the Line of Numbers for AC.

CASE 2.

Two Angles and a Side given, to find the other Sides.

In the Triangle ABC, there is the Angle A 46°. 30', *AB* 230, *and the Angle B* 37. 30 *given, to find AC and BC. Fig.* 15.

Geometrically.

Draw a blank Line, upon which set AB 230, from a Scale of equal Parts; at the Point A of the Line AB, make an Angle of 46°. 30', by a blank Line; and at the Point B of the Line AB, make an Angle of 37°. 30', by another blank Line; the Intersection of those Lines gives the Point C, so is your Triangle ABC constructed. Measure AC and BC from the same Scale of equal Parts that AB was taken; and you have the Answer required.

By Calculation.

By (Cor. 1. Theo. 5. Section 1.) 180—the Sum of the Angles A and B = C.

Plate V.

$$\begin{array}{ll} A & 46^\circ.\ 30' \\ B & 37.\ 30 \\ \hline \end{array}$$

$$180^\circ - \text{———} 84.\ 00 = 96^\circ.\ 00' = C$$

By Def. 29. Sect. 1. The Sine of 96° = the Sine of 84°, which is the Supplement thereof; therefore instead of the Sine of 96°, look in the Tables for the Sine of 84°.

By Theo. 1. of this Sect.

As the Sine of C	96°. 00'	9.99761
is to AB	230	2.36173
So is the Sine of A	46°. 30'	9.86056
		12.22229
To BC	167.8	2.22468

As the Sine of C	96°. 00'	9.99761
is to AB,	230	2.36173
So is the Sine of B	37°. 30'	9.78445
		12.14618
to AC,	140 8	2.14857

By Gunter's *Scale.*

Extend from 84°, (which is the Supplement of 96°,) to 46°. 30', on the Sines; that Distance will reach from 230, to 168, on the Line of Numbers for BC.

Extend

Plate V.

Extend from 84°, to 37°. 30, on the Sines; that Extent will reach from 230, to 141, on the Line of Numbers for AC.

C A S E 3.

Two Sides, and a contained Angle given; to find the other Angles and Side.

In the Triangle ABC, there is AB 240, *the Angle A* 36°. 40', *and AC* 180 *given; to find the Angles C and B, and the Side BC. Fig.* 16.

Geometrically.

Draw a blank Line, on which from a Scale of equal Parts, lay AB 240: at the Point A of the Line AB, make an Angle of 36°. 40', by a blank Line; on which from A, lay AC 180, from the fame Scale of equal Parts; Meafure the Angles C, and B, and the Side BC, as before; and you have the Anfwer required.

By Calculation.

By Cor. 1. Theo. 5, Sect. 1. 180—the Angle A 36°. 40', = 143°. 20', the Sum of the Angles C, and B: Therefore half of 143°. 20', will be half the Sum of the two required Angles, C, and B.

By Theo. 2, of this Sect.

As the Sum of the two Sides AB, and AC, 420
is to their Difference, 60

So

Plate V.

So is the Tangent of half the Sum of the two unknown Angles C and B	$71^{\circ}.\ 40'$
to the Tangent of half their Difference	$23^{\circ}.\ 20'$

By Theo. 4.

To half the Sum of the Angles C and B	$71^{\circ}.\ 40'$
Add half their Difference as now found	23. 20
The Sum is the greatest Angle, or Ang. C	95. 00
Subtract, and you have the least Angle, or B	48. 20

The Angles C, and B, being found; BC is had as before, by Theo. 1. of this Sect. Thus.

S. B : AC : : S : A : BC
$48^{\circ}.\ 30'$ 180 $36^{\circ}.\ 40'$ 143. 9

By Gunter's *Scale.*

Because the two first Terms are of the same Kind, extend from 420 to 60 on the Line of Numbers; lay that Extent from 45° on the Line of Tangents, and keeping the left Leg of your Compasses fixed, move the right Leg to $71^{\circ}.\ 40'$; that Distance laid from 45° on the same Line, will reach to $23^{\circ}\ 30'$, the half Difference of the required Angles. Whence the Angles are obtained as before.

The second Proportion may be easily extended, from what has been already said.

CASE

Plate V.

CASE 4.

The Sides given, to find the Angles.

In the Triangle ABC, there is given AB 64, *AC* 47, *BC* 34: *The Angles A, B, C, are required.* Fig. 17.

Geometrically.

The Construction hereof must be manifest, from Prob. 1. Sect. 1.

By Calculation.

From the Point C, let fall the Perpendicular CD, on the Base AB; it will divide the Triangle into two right-angled ones, ADC, and CBD; as well as the Base AB, into the two Segments, AD and DB.

AC	47
BC	34
Sum	81
Difference	13

By Theo. 3, of this Sect.

As the Base or the longest Side, AB,	64
is to the Sum of the other Sides AC and BC,	81
So is the Difference of those Sides,	13
to the Difference of the Segments of the Base AD, DB, - - - -	16.46

By

Plate V.

By Theo. 4, of this Sect.

To half the Base, or to half the Sum of the Segments AD and DB,	32
Add half their Difference, now found,	8.23
Their Sum will be the greatest Segment AD,	40 23
Subtract, and their Difference will be the least Segment DB, ----	23.77

In the right-angled Triangle ADC; there is AC 47, and AD 40. 23, given, to find the Angle A.

This is resolved by Case 4, of right-angled plane Trigonometry, thus,

AD : R : : AC : Sec. A.
$40.\ 23 : 90^{\circ} : : 47 : 31^{\circ}.\ 08^{\prime}$

Or it may be had by finding the Angle ACD, the Complement of the Angle A; without a Secant, thus,

AC : R : : AD : S. ACD.
$47 : 90^{\circ} : : 40.\ 23 : 58^{\circ}.\ 52^{\prime}$

$90-58^{\circ}.\ 52^{\prime}=31^{\circ}.\ 08^{\prime}$, the Angle A.

Then, by Theo. 1. of this Sect.

BC : S. A : : AC : S. B.
$34 : 31^{\circ}.08^{\prime} : : 47 : 45^{\circ}.\ 37^{\prime}$.

By

By Cor. 1. Theo. 5. Sect. 1. 180—the Sum of A and B, =C

A 31°. 08′
B 45. 37

180—76. 45=103°. 15′, the Angle C.

By Gunter's *Scale.*

The first Proportion is extended on the Line of Numbers; and it is no Matter whether you extend from the first to the third, or to the second Term, since they are all of the same Kind: If you extend to the second, that Distance applied to the third, will give the fourth; but if you extend from the first to the third, that Extent will reach from the second to the fourth.

The Methods of extending the other Proportions have been already fully treated of.

An Example in each Case of Oblique Angular Trigonometry.

1. Given, $\left\{\begin{array}{ll}AC & 290\\ C & 69°.\ 30'\\ AB & 350\end{array}\right\}$ $\begin{array}{l}A\\ B\\ BC\end{array}$ required.

2. Given, $\left\{\begin{array}{ll}C & 24°.\ 20'\\ B & 128°.\ 30'\\ AC & 3246\end{array}\right\}$ $\begin{array}{l}AB\\ \\ BC\end{array}$ required.

3. Given, $\left\{\begin{array}{ll}AC & 6\\ C & 124°.\ 30'\\ BC & 4.\ 5\end{array}\right\}$ $\begin{array}{l}A\\ B\\ AB\end{array}$ required.

Plate V.

4. Given, $\left\{\begin{matrix} AB & 46 \\ AC & 92 \\ BC & 52 \end{matrix}\right\}$ $\begin{matrix} A \\ B \\ C \end{matrix}$ required.

Having thus gone through Plane Trigonometry, we ſhall now proceed to apply the ſame, in determining the Meaſures of inacceſſible Heights and Diſtances. And firſt,

Of

Of HEIGHTS.

Plate V.

THE Inſtrument of leaſt Expence for taking Heights, is a Quadrant, divided into 90 equal Parts or Degrees; and thoſe may be ſubdivided into Halves, Quarters, or Eighths, according to the Radius, or Size of the Inſtrument: its Conſtruction will be evident by the Scheme thereof. (Fig. 18)

From the Center of the Quadrant let a Plummet be ſuſpended by a Horſe Hair, or a fine Silk Thread; of ſuch a Length that it may vibrate freely, near the Edge of its Arc: By looking along its Edge AC, to the Top of the Object whoſe Height is required; and holding it perpendicular, ſo that the Plummet may neither ſwing from it, nor lie on it; the Degree then cut by the Hair, or Thread, will be the Angle of Altitude required.

If the Quadrant be fixed upon a Ball and Socket, on a three-legged Staff, and if the Stem from the Ball be turned into the Notch of the Socket, ſo as to bring the Inſtrument into a perpendicular Poſition; the Angle of Altitude by this Means, can be acquired with much greater Certainty.

Plate V.

An Angle of Altitude may be also taken by any of the Instruments used in Surveying; as shall be particularly shewn, when we treat of their Descriptions and Uses.

Most Quadrants have a Pair of Sights fixed on the Edge AC, with small circular Holes in them; which are useful in taking the Sun's Altitude; requisite to be known in many astronomical Cases; this is effected by letting the Sun's Ray, which passes thro' the upper Sight fall upon the Hole in the lower one; and the Degree then cut by the Thread, will be the Angle of the Sun's Altitude; but those Sights are useless for our present Purpose, for looking along the Quadrant's Edge to the Top of the Object will be sufficient, as before.

PROB. I.

Plate 5. Fig. 19.
To find the Height of a perpendicular Object at one Station, which is on an horizontal Plane.

A Steeple.

Given, { The Angle of Altitude 53 Degrees.
Distance from the Observer to the Foot of the Steeple, or the Base, 85 Feet.
Height of the Instrument, or of the Observer, 5 Feet.

Required the Height of the Steeple.

The Figure is constructed and wrought, in all Respects as Case 2 of right angled Trigonometry; only there must be a Line drawn parallel to, and beneath

Plate V. Fig. 19.

beneath AB of 5 Feet for the Obſerver's Height, to repreſent the Plane upon which the Object ſtands; to which the Perpendicular muſt be continued, and that will be the Height of the Object.

Thus, AB is the Baſe, A the Angle of Altitude, BC the Height of the Steeple from the Inſtrument, or from the Obſerver's Eye, if he were at the Foot of it; DC the Height of the Steeple above the horizontal Surface.

Various Statings for BC, as in Caſe 2 of right angled Plane Trigonometry.

$$\begin{array}{l} 90^\circ \\ 53 = A \\ \hline 37 = C \\ \hline \end{array}$$

1. S. C : AB : : S. A : BC.
   37° 85 53° 112. 8.

2. R. : AB : : T. A : BC.
   90° 85 53° 112. 8.

3. T. C : AB : : R. : BC.
   37° 85 90° 112. 8

To BC,	112. 8	
Add DB,	5.	the Height of the Obſerver
Their Sum is	117. 8	or 118 Feet, the Height

of the Steeple required.

PROB.

PROB. II.

Plate V. Fig. 20.

To find the Height of a Perpendicular Object, on an horizontal Plane; by having the Length of the Shadow given.

Provide a Rod, or Staff, whose Length is given, let that be set perpendicular, by Help of a Quadrant, thus; apply the Side of the Quadrant AC, to the Rod, or Staff; and when the Thread cuts 90°, it is then perpendicular; the same may be done by a Carpenter's or Mason's Plumb.

Having thus set the Rod or Staff perpendicular; measure the Length of its Shadow, when the Sun shines, as well as the Length of the Shadow of the Object, whose Height is required; and you have the proper Requisites given. Thus,

ab, the Length of the Shadow of the Staff, 15 Feet.

bc, the Length of the Staff, 10 Feet.

AB, the Length of the Shadow of the Steeple, or Object, 135 Feet.

Required BC, the Height of the Object.

The Triangles *abc*, ABC, are similar, thus: The Angle *b*=B, being both right; the Lines *ac* AC, are parallel, being Rays, or a Ray of the Sun; whence the Angle *a*=A (by Part 3. Theo. 3. Sect. 1.) and consequently *c*=C. The Triangles being therefore

therefore mutually equiangular, are similar, (by Theo. 16. Sect. 1.) it will be,

ab : *bc* : : AB : BC.

15 10 135 90. the Steeple's Height required.

The foregoing Method is most to be depended on; however, this is mentioned for Variety sake.

PROB. III.

To take the Altitude of a Perpendicular Object, at the Foot of a Hill, from the Hill's Side.

Turn the Center A of the Quadrant next your Eye, and look along the Side AC, or 90 Side, to the Top and Bottom of the Object; and noting down the Angles, measure the Distance from the Place of Observation to the Foot of the Object, Thus,

Given, { Angle to the Foot of the Object $55^{\circ}\frac{1}{4}$ or $55^{\circ}.\ 15^{\prime}$.
Angle to the Top of it $31^{\circ}\frac{1}{4}$ or $31^{\circ}.\ 15^{\prime}$.
Distance to the Foot of it 250 Feet. }

Required the Height of the Object.

Geometrically.

Draw an infinite blank Line AD, at any Point in which A, make the Angle EAB of $55^{\circ}.\ 15^{\prime}$, and EAC of $31^{\prime}.\ 15^{\prime}$; lay 250 from A to B; from B draw the Perpendicular BE (by Prob. 7. of Geometry) crossing AC in C; so will BC be the Height of the Object required.

In

Plate V. Fig. 21.

In the Triangle ABC there is given,

ABE the Complement of EAB to 90°, which is $34^{\circ}. 45^{\prime}$.
ACB the Difference of the given Angles $24^{\circ}. 00^{\prime}$.
The Side AB 250. Required BC.

This is performed as Case 2, of Oblique Angular Trigonometry. Thus,
180—the Sum of ABE $34^{\circ}. 45^{\prime}$, and CAB $24^{\circ}. 00^{\prime}$ $=$ACB $121^{\circ}. 15^{\prime}$. Then,

S. ACB : AB : : S. CAB : BC.
$121^{\circ}. 15^{\prime}$ 250 $24^{\circ}. 00^{\prime}$ 119. The Height required.

PROB. IV.

To take the Altitude of a perpendicular Object, on the Top of a Hill, at one Station; when the Top and Bottom of it can be seen from the Foot of the Hill.

Plate V. Fig. 22.

As in Prob. 1; take an Angle to the Top, and another to the Bottom of the Object; and measure from the Place of Observation to the Foot of the Object, and you have all the given Requisites. Thus.

A Tower on A Hill.

Given, { Angle to the Bottom 48° $30^{\prime}$.
Angle to the Top $67^{\circ}. 00^{\prime}$.
Dist. to the Foot of the Object 136 Feet.
Required, the Height of the Object.

Geo-

Plate V. Fig. 22.

Geometrically.

Make the Angle DAB = 48°. 30', and lay 136 Feet from A to B; from B let fall the Perpendicular BD, and that will be the Height of the Hill: Produce BD upwards by a blank Line: Again, at A make the Angle DAC = 67°. 00' by a blank Line, and from C where that crosses the Perpendicular produced, draw the Line CB, and that will be the Height of the Object required.

Let AC be drawn.
In the Triangle ABC, there is given,
The Angle ACD the Complement of DAC = 23° 00'.
CAB the Difference between the two given Angles = 18°. 30'.
And the Side AB 136. To find BC.

S. C : AB : : S. CAB : BC.
23° 136 18°. 30' $110\frac{1}{2}$

If BD were wanted, it is easily obtained, by the first Case of Right-angled Plane Trigonometry.

PROB. V.

To take an inaccessible perpendicular Altitude, on an horizontal Plane.

Plate V. Fig. 23.

This is done at two Stations, thus:

Plate V. Fig. 23.

Let DC be a Tower which cannot be approached by means of a Mote or Ditch nearer than B; at B, take an Angle of Altitude to C: Measure any convenient Distance backward to A, which note down: At A take another Angle to C; so have you the given Requisites, thus;

Given, $\begin{cases} \text{First Angle } 55^{\circ}.\ 00' \\ \text{Stationary Distance 87 Feet.} \\ \text{Second Angle } 37^{\circ}.\ 00'. \end{cases}$

The Height of the Tower CD, is required.

Geometrically.

Upon an infinite blank Line, lay off the stationary Distance 87, from A to B; from B, set off your first; and from A, your second Angle: From C, the Point of Intersection of the Lines which form these Angles, let fall the Perpendicular CD; and that will be the Height of the Object required.

The external Angle CBD, of the Triangle ABC; is equal to the two internal opposite ones, A, and ACB (by Theo. 4. Sect. 1.): Wherefore if one of the internal opposite Angles be taken from the external Angle; the Remainder will be the other internal opposite one, thus;

$$\text{CBD } 55^{\circ} - \text{A } 37^{\circ} = \text{ACB } 18^{\circ}.$$

Therefore in the Triangle ABC; we have the Angles A, and ACB, with the Side AB given; to find BC.

S. ACB

Plate V. Fig. 23.

S. ACB : AB : : S. A : BC.
18° 87 37° 169.4

Having found BC, we have in the Triangle BCD; the Angle CBD 55°, consequently BCD 35°, and BC 169.4; to find DC.

This is performed by Case the first, of Right-angled Trigonometry, three several Ways; thus,

1. R : BC : : S. CBD : DC.
90° 169.4 55° 138.8. The Height required.

2. Sec. CBD : BC : : T. CBD : DC.
55° 169.4 55° 138.8.
The Height required.

3. Sec. BCD : BC : : R : CD.
35° 169.4 90° 138.8.
The Height required.

If BD, the Breadth of the Mote, were required; it may also be found, by three different Statings, as in the first Case of Right-angled Plane Trigonometry.

PROB. VI.

Plate V. Fig. 24.

Let BC a May-pole, whose Height is 100 Feet, be broken at D; the upper Part of which DC, falls upon an horizontal Plane, so that its extremity C, is 34 Feet from the Bottom or Foot of the Pole.

Required the Segments BD and DC.

Geometrically.

Lay 34 Feet from A to B; on B erect the Perpendicular BC of 100 Feet, and draw AC: Bisect AC, (by Prob. 4. Sect. 1.) with the Perpendicular Line EF; and from D, where it cuts the Perpendicular BC, draw AD, which will be the upper Segment; and DB will be the lower.

By Cor. to Lemma, preceding Theo. 7. Sect. 1. AD = DC; and (by the Lemma) the Angle C = CAD.

In the Triangle ABC, find C, as in Case 6, of Right-angled Trigonometry, thus;

1. BC : R : : AB : T. C = GAD.
   100 90° 34 $18^{\circ}.\ 47^{1}$

By Theo. 4. Sect. 1. The external Angle ADB = $37^{\circ}.\ 34^{1}$, or to twice the Angle C; i. e. to C, and GAD.

Then in the Triangle ABD, there is ADB $37^{\circ}.\ 34^{1}$, therefore also its Complement DAB $52^{\circ}.\ 26^{1}$, and AB 34 given, to find AD, and AB.

By the second Case of Rectangular Trigonometry.

2. S. ADB : AB : : R : AD, or DC.
   $37.\ 34^{1}$ 34 90° 55.77.

   BC—DC=BD.
   100—55.77=44.23 required.

These

These may be had from other Statings, as in the second Case aforesaid.

PROB. VII.

To take the Altitude of a perpendicular Object on a Hill, from a Plane beneath it.

Plate V. Fig. 25.

This is done at two Stations, thus;

Let the Height DC, of a Wind-mill on a Hill be required.

From any Part of the Plane whence the Foot of the Object can be seen, let Angles be taken to the Foot, and Top; measure thence any convenient Distance towards the Object, and at the End thereof, take another Angle to the Top: And you have the proper Requisites, thus;

First Station. Angle to the Foot DAB $21^{\circ}.\ 00'$.
Angle to the Top CAB $35^{\circ}.\ 00'$.
Stationary Distance AB 104 Feet,

Second Station. Angle to the Top $48^{\circ}.\ 30'$.

DC, and DE required.

Geometrically.

On an infinite blank Line, lay the stationary Distance AB 104 Feet; from A, set off the second, and from B, the third given Angle; and from the

int-

Plate V. Fig. 25.

interfecting Point C of the Lines formed by them; let fall the Perpendicular CE: From A fet off the firft Angle, and the Line formed by it will determine the Point D. Thus have we the Height of the Hill, as well as that of the Windmill.

The Angle CBE—A=ACB, as before.

In the Triangle ABC, find BC thus,

1. S. ACB : AB : : S. A : BC.
$13^{\circ}.\ 30^{\prime}$ 104 $35^{\circ}.\ 00^{\prime}$ 255.5.

In the Triangle CBE, find CE thus,

2. R. : BC : : S. CBE : CE.
90° 255.5 $48^{\circ}.\ 30^{\prime}$ 191.4.

In the fame Triangle find BE thus,

3. R. : BC : : S. BCE : (or Comp. of CBE) BE.
90° 255.5 $41^{\circ}.\ 30^{\prime}$ 169.3.

AB+BE=AE, or 104+169.3=273.3=AE.

In the Triangle AED, find DE thus,

4. S. ADE : (or Comp. of DAB) AE : : S. DAE
69° 273.3 21
: DE.
104.9, required.

CE—DE=CD, or 191.4—104.9=86.5=CD, required.

CE,

Plate V. Fig. 25.

CE, BE, or DE, may be found by other various Statings, as set forth in the first and second Cases of Rectangular Trigonometry.

PROB. VIII.

To find the Length of an Object, that stands obliquely on the Top of a Hill, from a Plane beneath.

Let CD be a Tree whose Length is required.

This is done at two Stations.

Make a Station at B, from whence take an Angle to the Foot, and another to the Top of the Tree; measure any convenient Distance backward to A, from whence also let an Angle be taken to the Foot, and another to the Top; and you have the Requisites given. Thus,

First Station.	Angle to the Foot EBD $= 36^{\circ}. 30'$
	Angle to the Top EBC $= 44^{\circ}. 30'$
Stationary	Distance AB 104 = Feet.
Second Station.	Angle to the Foot EAD $= 24^{\circ}. 30'$.
	Angle to the Top EAC $= 32^{\circ}. 00'$.

Let DC and DE be required.

The Geometrical Constructions of this and the next Problem are omitted; as what has been already said, and the Figures are looked upon as sufficient Helps.

EBC—A = ACB, or $44^{\circ}. 30' - 32^{\circ} = 12^{\circ}. 30'$, as before.

In

Plate V. Fig. 26.

In the Triangle ABC, find BC. Thus:

1. S. ACB : AB : : S. A : BC.
12° 30′ 104 32° 254.7.

EBD—EAD=ADB, or 36°. 30′—24°. 30′= 12°. 00′.

In the Triangle ADB, find BD. Thus:

2. S. ADB : AB : : S. DAB : DB.
12°. 00′ 104 24°. 30′ 207. 4

CBE—DBE = CBD, or 44°. 30′ — 36°. 30′ = 8°. 00′.

In the Triangle CBD there is given, CB 254.7, DB 207.4, and the Angle CBD 8°. 00′; to find DC.

This is performed as Case 3, of Oblique angular Trigonometry, thus:

3. BC+BD : BC—BD : : T. of $\frac{1}{2}$ BDC+BCD :
462.1. 47.3 86° 00′.
T. of $\frac{1}{2}$ BDC—BCD.
55°. 40.

86°. 00+55°. 40′ = 141°. 40′ = BDC.
86°. 00′—55°. 40′=30°. 20′ = BCD.

4. S. BCD : BD : : S. CBD : DC.
30°. 20′ 207.4 8. 00′ 57.15 Length of the Tree.

To find DE, in the Triangle DBE.

Say R. : BD : : S. DBE : DE.
90°. 207.4. 36° 30′ 123.4. Height, of the Hill.

PROB.

PROB. IX.

To find the Height of an inacceſſible Object CD, on a Hill BC, from Ground that is not horizontal.

Plate VI. Fig. 1.

From any two Points, as G and A, whoſe Diſtance GA is meaſured, and therefore given; let the Angles HGD, BAD, BAC, and EAG, be taken: Becauſe GH is parallel to EA (by Part 2. Theo. 3. Sect. 1.) the Angle HGA=EAG; therefore EAG +HGD=AGD: And (by Cor. 1. Theo. 1. Sect. 1.) 180 — the Sum of EAG and BAD=GAD: And (by Cor. 1. Theo. 5. Sect 1.) 180—the Sum of the Angles AGD and GAD=GDA: Thus we have the Angles of the Triangle AGD, and the Side AG given; thence (by Caſe 2, of Ob. Trig.) AD may be eaſily found. The Angle DAB—CAB =DAC, and 90°—BAD=ADC; and 180°—the Sum of DAC and ADC=ACD: So have we the ſeveral Angles of the Triangle ACD given, and the Side AD; whence (by Caſe 2, Obl. Trig.) CD may be eaſily found. We may alſo find AC, which with the Angle BAC, will give CB the Height of the Hill.

The Solutions of the ſeveral Problems in Heights and Diſtances, by *Gunter*'s *Scale*, are omitted; becauſe every particular Stating has been already ſhewn by it, in the Rectangular and Obliqueangular Trigonometry.

Of DISTANCES.

ANY of the Inſtruments uſed in Surveying, will give you the Angles or Bearings of Lines; which will be particularly ſhewn, when we come to treat of them.

PROB. I.

Plate VI. Fig. 2.

Let A and B be two Houſes on one Side of a River, whoſe Diſtance aſunder is 293 Perches: There is a Tower at C on the other Side of the River, that makes an Angle at A, with the Line AB of $53^{\circ}.\ 20^{\prime}$; and another at B, with the Line BA of $66^{\circ}.\ 20^{\prime}$: Required the Diſtance of the Tower from each Houſe, *viz.* AC and BC.

This is performed as Caſe 2, of Oblique-angled Trigonometry, thus:

1. S. C : AB : : S. A : BC.
   $60^{\circ}.\ 20^{\prime}$ 293 $53^{\circ}.\ 20^{\prime}$ 270. 5.

2. S. C : AB : : S. B : AC.
   $60^{\circ}.\ 20^{\prime}$ 293 $66^{\circ}.\ 20^{\prime}$ 308. 8.

PROB.

PROB. II.

Plate VI. Fig. 11.

Let B, and C, be two Houses whose direct Distance asunder BC, is inaccessible: However it is known that a House at A, is 252 Perches from B, and 230 from C; and that the Angle BAC, is found to be 70°. What is the Distance BC, between the two Houses?

This is performed as Case 3, of Oblique-angular Trigonometry, thus:

1. $AB+AC : AB-AC :: T.\ of\ \frac{1}{2}\ C+B :$
   482 22 55°. 00′

   $T.\ of\ \frac{1}{2}\ C-B$
   3°. 44′.

$55^\circ + 3^\circ.\ 44' = 58^\circ\ 44' = C.$ $55^\circ - 3^\circ.\ 44' = 51^\circ.\ 16' = B$

2. $S.\ C : AB :: S.\ A : BC.$
   58°.44′ 252 70° 279.9

PROB. III.

Plate VI. Fig. 3.

Suppose ABC a triangular Piece of Ground, which by an old Survey we find to be thus: AB 260, AC 160, BC 150 Perches: The mearing Lines AC, and BC, are destroyed or plowed down,

 and

ACD. The Triangles CEB, CAD, are therefore mutually equiangular, and (by Theo. 16. Sect. 1.) DC : AC : : CB : CE, or DC : CB : : AC : CE. Q. E. D.

PROB. V.

Plate VI. Fig. 5.

Let three Gentlemens Seats, A, B, C, be ſituate in a triangular Form: There is given AB 2. 5, Miles, AC 2. 3, and BC 2. It is required to build a Church at E, that ſhall be equi-diſtant from the Seats A, B, C. What Diſtance muſt it be from each Seat, and by what Angle may the Place of it be found?

Geometrically.

By Prob. 15. Sect. 1. Find the Center of a Circle that will paſs through the Points A, B, C; and that will be the Place of the Church; the Meaſure of which, to any of theſe Points, is the Anſwer for the Diſtance: Draw a Line from any of the three Points to the Center, and the Angle it makes with either of the Sides that contain the Angle it was drawn to; that Angle layed off by the Direction of an Inſtrument, on the Ground, and the Diſtance before found being ranged thereon, will give the Place of the Church required.

By Calculation.

1.	AB :	AC+BC : :	AC−BC :	AD−DB.
	2.5	4.3	.3	.516.

$1.25 + .258 = 1.508 = AD.$

By

By Cor. 2. Theo. 14. Sect. 1. The Square Root of the Difference of the Squares of the Hypothenuse AC, and given Leg AD, will give DC.

i. e. $5.29 - 2.274064 = 3.015936$.

Its Square Root is $1.736 = CD$.

Then by the preceding Lemma,

2.	CD :	AC :	: CB :	the Diameter.
	1.736	2.3	2	2.65.

the half of which, *viz.* 1.325 is the Semi-diameter, or Distance of the Church from each Seat, that is AE, CE, BE.

From the Center E, let fall a Perpendicular upon any of the Sides as EF, and it will bisect in E: (by Theo. 8. Sect. 1.)

Wherefore $AF = CF = \frac{1}{2} AC = 1.15$.

In the right-angled Triangle AFE, you have AF 1.15, and AE the Radius 1.325 given, to find FAE, thus,

3.	AF :	R. :	: AE :	Sec. FAE.
	1.15	90°	1.325	$29^{\circ}.\ 47^{l}$.

Wherefore directing an Instrument to make an Angle of $29^{\circ}.\ 47^{l}$, with the Line AC; and measuring 1.325 on that Line of Direction, will give the Place of the Church, or the Center of a Circle that will pass through A, B, and C.

The

The above Angle FAE, may be had without a Secant as before, thus,

AE : R. : AF : S. AEF.
1·325 90° 1·15 60°. 13'.

Its Complement 29°. 47', will give FAE, as before.

The Questions that may be proposed on this Head, being innumerable, we have chosen to give only a few of the most useful.

Of

SECT. III.

Containing a particular Description of the several Instruments used in Surveying, with their respective Uses. And first,

Of the CHAIN.

THE Stationary Distance, or Mearings of Ground, are measured either by Mr. *Gunter*'s Chain of four Poles or Perches, which consists of 100 Links; (and this is the most natural Division) or by one of 50 Links, which contains two Poles or Perches: But because the Length of a Perch differs in many Places, therefore the Lengths of Chains and their respective Links will differ also.

The *English Statute-Perch* is $5\frac{1}{2}$ Yards, the Two-Pole-Chain is 11 Yards, and the Four-Pole one is 22 Yards: Hence the Length of a Link in a *Statute-Chain* is 7.92 Inches.

There are other Perches used in different Parts of *England*, as the Perch of *Wood-land Measure*, which is 6 Yards; that of *Church-land Measure*, which is 7 Yards (or the same with the *Plantation Perch*) and the *Forest Measure Perch*, which is 8 Yards.

The *Irish*, or *Plantation Perch* is 7 Yards, as before; the Two-Pole Chain is 14; and the Four-Pole one is 28 Yards: Hence the Length of a *Plantation Chain* is 10.08 Inches.

The *Scotch Perch* is $18\frac{1}{2}$ Feet, or $6\frac{1}{6}$ Yards, or 6 *Scot's* Ells. In the Shire of *Cunningham* in *Scotland*, their Perch is $18\frac{3}{4}$ Feet, and this Perch is used in some few Places in the North Part of this Kingdom, as the Statute Perch is in some other Parts.

For the more ready reckoning the Links of a Four-Pole Chain, there is a large Ring, or sometimes a round Piece of Brass fixed at every 10 Links; and at 50 Links, or in the Middle, there are 2 large Rings. In such Chains as have a Brass Piece at every 10 Links, there is the Figure 1 on the first Piece, 2 on the second, 3 on the third, *&c.* to 9. By leading therefore that End of the Chain forward, which has the least Number next it, he who carries the hinder End may easily determine any Number of Links: Thus, if he has the Brass Piece Number 8, next to him, and 6 Links more in a Distance, that Distance is 86 Links. After the same Manner 10 may be counted for every large Ring of a Chain which has not Brass Pieces on it; and the Number of Links is thus readily determined.

The Two-Pole Chain has a large Ring at every 10 Links, and in its Middle, or at 25 Links, there are 2 large Rings; so that any Number of Links may be the more readily counted off, as before.

The Surveyor should be careful to have his Chain measured before he proceeds on Business, for the Rings

Rings are apt to be open by frequent using it, and its Length is thereby encreased, so that no one can be too circumspect in this Point.

In measuring a stationary Distance, there is an Object fix'd in the extream Point of the Line to be measured; this is a Direction for the hinder Chainman to govern the foremost one by, in order that the Distance may be measured in a right Line; for if the hinder Chainman causes the other to cover the Object, it is plain the foremost is then in a right Line towards it. For this Reason it is necessary to have a Person that can be relied on, at the hinder End of the Chain, in order to keep the foremost Man in a right Line; and a Surveyor who has no such Person should chain himself. The Inaccuracies of most Surveys arise from bad Chaining, that is, from straying out of the right Line, as well as from other Omissions of the hinder Chainman: No Person, therefore, should be admitted at the hinder End of the Chain, of whose Abilities in this respect, the Surveyor was not previously satisfied and convinced; since the Success of the Survey in a great Measure depends on his Care and Skill.

In setting out to measure any stationary Distance, the fore Man of the Chain carries with him 10 Iron Pegs pointed, each about ten Inches long; and when he has stretched the Chain to its full Length, he at the Extremity thereof sticks one of those Pegs perpendicularly in the Ground; and leaving it there, he draws on the Chain 'till the hinder Man checks him when he arrives at that Peg: The Chain being again stretched, the fore Man sticks down another Peg, and the hind Man takes up the former; and thus they proceed at every Chain's Length contained in the Line to be measured, counting the surplus Links contained between the

last Peg, and the Object at the Termination of the Line, as before: So that the Number of Pegs taken up by the hinder Chainman, expresses the Number of Chains; to which, if the odd Links be annexed, the Distance Line required in Chains and Links is obtained, which must be register'd in the Field-Book, as will hereafter be shewn.

If the Distance exceeds 10, 20, 30, &c. Chains, when the Leader's Pegs are all exhausted, the hinder Chainman, at the Extremity of the 10 Chains, delivers him all the Pegs; from whence they proceed to measure as before, 'till the Leader's Pegs are again exhausted, and the hinder Chainman at the Extremity of these 10 Chains again delivers him the Pegs; from whence they proceed to measure the whole Distance Line in the like Manner; then tis plain, that the Number of Pegs the hinder Chainman has, being added to 10, if he had delivered all the Pegs once to the Leader, or to 20 if twice, or to 30 if thrice, &c. will give the Number of Chains in that Distance; to which if the surplus Links be added, the Length of the stationary Distance is known in Chain and Links.

It is customary, and indeed necessary, to have Red, or other coloured Cloth fixed to the Top of each Peg, that the hinder Man at the Chain may the more readily find them; otherwise in chaining thro' Corn, high Grass, Briars, Rushes, Potatoes, &c. it would be extremely difficult to find the Pegs which the Leader puts down: By this Means no Time is lost, which otherwise must be, if no Cloths are fixed to the Pegs, as before.

It will be necessary here to observe, that all slant, or inclined Surfaces, as Sides of Hills, are measured horizontally, and not on the Plane or Surface of the Hill, and is thus effected: Let

Plate VIII. Fig. 4.

Let ABC be a Hill, the hindmost Chainman is to hold the End of the Chain perpendicularly over the Point A, (which he can the better effect with a Plummet and Line, than by letting a Stone drop, which is most usual) as d is over A, while the Leader puts down his Peg at e: The Eye can direct the horizontal Position near enough, (but if greater Accuracy were required, a Quadrant applied to the Chain, would settle that). In the same Manner the rest may be chained up and down; but in going down it is the plain Leader of the Chain must hold up the End thereof, and the Plummet thence suspended will mark the Point where he is to stick his Peg. The Figure is sufficient to render the whole evident; and to shew that the Sum of the Chains will be the horizontal Measure of the Base of the Hill; for $de = Ao$, $fg = op$, $hi = pq$, &c. therefore $de + fg + hi$, &c. $= Ao + op + pq$, &c. $=$ AC, the Base of the Hill. If an whole Chain cannot be carried horizontally, half a one or less may, and the Sum of these half Chains, or Links, will give the Base as before.

If the inclined Side of the Hill be a plane Surface, the Angle of the Hill's Inclination may be taken, and the slant Height may be measured on the Surface; and thence (by Case 1, of right-angled Trigonometry,) the horizontal Line answering to the Top may be found; and if we have the Angle of Inclination given on the other Side, with those already given; we can find the horizontal Distance across the Hill, by Case 2, of Oblique Trigonometry.

All inclined Surfaces are considered as horizontal ones; for all Trees which grow upon any inclined Surface,

Surface, do not grow perpendicular thereto, but to the Plane of the Horizon: Thus if A*d*, *ef*, *gh*, &c. were Trees on the Side of a Hill, they grow perpendicular to the horizontal Baſe AC, and not to the Surface AB: Hence the Baſe will be capable to contain as many Trees as are on the Surface of the Hill, which is manifeſt from the Continuation of them thereto. And this is the Reaſon that the Area of the Baſe of a Hill is conſidered to be equal in Value to the Hill itſelf.

Beſides, the Irregularities of the Surfaces of Hills in general are ſuch, that they would be found impoſſible to be determined by the moſt able Mathematician. Certain regular curve Surfaces have been inveſtigated with no ſmall Pains by the moſt Eminent: Therefore an Attempt to determine in general the Infinity of irregular Surfaces which offer themſelves to our View, to any Degree of Certainty, would be idle and ridiculous, and for this Reaſon alſo the horizontal Area is only attempted.

Again, if the circumjacent Lands of a Hill be planned or mapp'd, it is evident we ſhall have a Plan of the Hill's Baſe in the Middle: But were it poſſible to put the Hill's Surface in lieu thereof, it would extend itſelf into the circumjacent Lands; and render the Whole an Heap of Confuſion: So that if the Surfaces of Hills could be determined, no more than the Baſe could be mapp'd.

Roads are uſually meaſured by a Wheel for that Purpoſe, to which there is fixed a Machine, at the End whereof there is a Spring, which is ſtruck by a Peg in the Wheel once in every Rotation; by this Means the Number of Rotations is known.

If ſuch a Wheel were 3 Feet 4 Inches in Diameter, one Rotation would be 10$\frac{1}{2}$ Feet, which is half a Plantation Perch; and becauſe 320 Perches make a Mile, therefore 640 Rotations will be a Mile alſo: And the Machinery is ſo contrived, that by Means of a Hand, which is carried round by the Work, it points out the Miles, Quarters, and Perches, or ſometimes the Miles, Furlongs, and Perches.

Or Roads may be meaſured by a Chain more accurately; for 80 Four-Pole, 160 Two-Pole Chains, or 320 Perches make a Mile as before: And if Roads are meaſured by a Statute Chain, it will give you the Miles *Engliſh*, but if by a Plantation Chain, the Miles will be *Iriſh*. Hence an *Engliſh* Mile contains 1760, and an *Iriſh* Mile 2240 Yards; and becauſe 14 half Yards is an *Iriſh*, and 11 half Yards is an *Engliſh* Perch, therefore 11 *Iriſh* Perches, or *Iriſh* Miles, are equal to 14 *Engliſh* ones.

Since ſome Surveys are taken by a Four-Pole, and others by a Two-Pole Chain; and as Ground for Houſes is meaſured by Feet, we will ſhew how to reduce one to the other, in the following Problems.

PROB. I.

To reduce Two-Pole Chains and Links to Four-Pole ones.

If the Number of Chains be even, the half of them will be the Four-Pole ones, to which annex the Links given, thus,

Ch. L.
1. In 16. 37. of Two-Pole Chains, how many Four-Pole ones?

Ch. L.
Anſwer, 8. 37.

But if the Number of Chains be odd, take the half of them for Chains, and add 50 to the Links, and they will be Four-Pole Chains and Links, thus,

Ch. L.
2. In 17. 42 of Two-Pole Chains, how many Four-Pole ones?

Ch. L.
Anſwer, 8. 92.

PROB. II.

To reduce Four-Pole Chains and Links, to Two-Pole ones.

Double the Chains, to which annex the Links, if they be leſs than 50; but if they exceed 50, double the Chains, add 1 to them, and take 50 from the Links, and the Remainder will be the Links, thus,

Ch. L.
1. In 8. 37 of Four-Pole Chains, how many
2 Two-Pole ones?

16. 37 Anſwer.

2. In

	Ch.	L.	
2. In	8.	82	of Four-Pole Chains, how many
	2.	50	Two Pole ones?
	17.	32	Anſwer.

PROB. III.

To reduce Four-Pole Chains and Links, to Perches and Decimals of a Perch.

The Links of a Four-Pole Chain are decimal Parts of it, each Link being the hundredth Part of a Chain; therefore if the Chains and Links be multiplied by 4, (for 4 Perches are a Chain) the Product will be the Perches and decimal Parts of a Perch. Thus,

	Ch.	L.	
How many Perches in	13.	64	of Four-Pole Chains?
		4	
Anſwer	54.	56	Perches.

PROB. IV.

To reduce Two-Pole Chains and Links, to Perches and Decimals of a Perch.

They may be reduced to Four-Pole ones, (by Prob. 1.) and thence to Perches and Decimals, (by the laſt.) or,

If the Links be multiplied by 4, carry one to the Chains, when the Links are, or exceed 25; and the Chains by 2, adding one if Occaſion be: The Product will be Perches, and Decimals of a Perch. Thus,

	C.	L.	
1. In	17.	21	of Two-Pole Chains, how many
	2.	4	Perches?
Anſwer	34.	84	Perches?

	C.	L.	
2. In	15.	38	of Two-Pole Chains, how many
	2.	4	Perches?
Anſwer	31.	52	Perches.

PROB. V.

To reduce Perches, and Decimals of a Perch, to Four-Pole Chains and Links.

Divide by 4, and cut two Decimals from the Quotient, and that will be Four-Pole Chains and Links. Thus,

In 31.52 Perches, how many Four-Pole Chains and Links?

C. L.
4)31.52(7. 88 Anſwer.

28
———
35
32
———
32
32
———

PROB.

PROB. VI.

To reduce Perches and Decimals of a Perch, to Two-Pole Chains and Links.

The Perches may be reduced to Four-Pole Chains, (by the laſt) and from thence to Two-Pole Chains, (by Prob. 2.) or,

Divide the whole Number by 2, the Quotient will be Chains; to the Remainder annex the given Decimals, and divide by 4, the laſt Quotient will be the Links. Thus,

In 31.52 Perches, how many Two-Pole Chains and Links?

```
              C.  L.
2)31.52( 15.  38.  Anſwer.
  ------
   11
  ------
4) 152 (38
  ------
    32
  ------
```

PROB. VII.

To reduce Chains and Links, to Feet and Decimal Parts of a Foot.

If they are Two-Pole Chains, reduce them to Four-Pole ones: (by Prob. 1.) Theſe being multi-

 plied

plied by the Feet in a Four-Pole Chain; will give the Feet, and Decimals of a Foot. Thus,

C. L.
In 17. 21 of Two-Pole Plantation-Chains, how many Feet?

```
        C.  L.
        8.  71 of Four-Pole Chains.
            84
        ------
          3484              Feet  Inches.
         6968      Answer   731.   7½
        ------
Feet    731.64
            12
        ------
Inches     7.68
              4
        ------
           2.72
        ------
```

PROB. VIII.

To reduce Feet and Inches to Chains and Links.

Reduce the Inches to the Decimal of a Foot, and annex that to the Feet; that divided by the Feet in a Four-Pole Chain, will give Four-Pole Chains and Links in the Quotient: These may be reduced to Two-Pole Chains and Links, if required, by Prob. 2. Thus,

Feet Inches
In 217. 9 how many Two-Pole Plantation-Chains?

```
12)9.00(.75 the Decimal of 9 Inches.
   ----
    60
   ----
```

```
        C.   L.
84)217·75(2.  59 of Four-Pole Chains.
   ———————
    497
   ————— C.  L.
    775 or 5. 09 of two Pole Chains.
   —————
     19
   —————
```

How to take a Survey by the CHAIN *only.*

PROB. I.

To ſurvey a Piece of Ground, by going round it, and the Method of taking the Angles of the Field, by the Chain only.

Plate VI. Fig. 5.

Let ABCDEFG be a Piece of Ground to be ſurveyed: beginning at the Point A, let one Chain be laid in a direct Line from A towards G, where let a Peg be left as at *c*, and again, the like Diſtance from A in a direct Line towards B, where another Peg is alſo to be left as at *d*: Let the Diſtance from *d* to *c* be meaſured, and placed in the Field-Book, in the ſecond Column under the Denomination of Angles, in a Line with Station No. 1; and in the ſame Line under the Title of Diſtances, in the third Column, let the Meaſure of the Line AB in Chains and Links be inſerted. Being now arrived at B, let one Chain be laid in a direct Line from B towards A, where let a Peg be left, at *f*,

and again, the like Diſtance from B in a direct Line towards C, where let alſo another Peg be left, as at *e*; the Diſtance from *e* to *f* is to be inſerted in the Field-Book, in the ſecond Column, under Angles, in a Line with Station, No. 2; and in the ſame Line, under the Title of Diſtances in the third Column, let the Meaſure of the Line BC, in Chains and Links, be inſerted: After the ſame Manner we may proceed from C to D, and thence to E; but becauſe the Angle at E, *viz.* FED, is an external Angle, after having laid one Chain from E to *h*, and to *g*, the Diſtance from *g* to *h* is meaſured, and inſerted in the Column of Angles, in a Line with Station, No. 5. and on the Side of the Field-Book againſt that Station, we make an Aſteriſk thus*, or any other Mark, to ſignify that to be an external Angle, or one meaſured out of the Ground. Proceed we then as before from E to F, to G, and thence to A, meaſuring the Angles and Diſtances, and placing them as before in the Field-Book oppoſite to their reſpective Stations; ſo will the Field-Book be compleated in Manner following.

N. B. After this Manner the Angles for inacceſſible Diſtances may be taken, and the Method of conſtructing or laying them down, as well as the Conſtruction of the Map, from the following Field Notes, muſt be obvious from the Method of taking them.

The Form of the Field-Book, with the Title.

A Field-Book of Part of the Land of *Grange*, in the Pariſh of *Portmarnock*, Barony of *Coolock*, and County of *Dublin*; being Part of the Eſtate of *L. P.* Eſq; ſet to *C. D.* Farmer. Surveyed *January* 30, 1768.

Taken

Taken by a Four-Pole Chain.

Remarks.	No. Sta.	Angles. C. L.	Diſtances C. L.
Mr. *J. D*'s Part of *Grange*	1	1.80	17.65
	2	1.79	18.50
Mr. *L. P*'s Part of *Portmarnock*	3	1.76	28.00
Strand	4	1.41½	20.00
*	5	1.87½	14.83
Widow *J. G*'s Part of *Grange*	6	1.14	19.41
	7	1.89	24.53

Cloſe at the 1ſt Station.

The Signification of the Remarks.

Mr. *J. D*'s Part of *Grange* mears, or is adjacent to the ſurveyed Land from the firſt to the third Station: Mr. *L. P*'s Part of *Portmarnock* mears it from the third to the fourth Station; the *Strand* then is the Mearing from thence to the ſixth, and from the ſixth to the firſt Station, the Widow *J. G*'s Part of *Grange* is the Mearing.

It is abſolutely neceſſary to inſert the Perſons Names, and Town-Lands, Strands, Rivers, Bogs, Rivulets, *&c.* which mear or circumſcribe the Land which is ſurveyed; for theſe muſt be expreſſed in the Map.

In a Survey of a Town-Land, or Eſtate, it is ſufficient to mention only the circumjacent Town-Lands, without the Occupiers Names; but when a Part only of a Town-Land is ſurveyed, then it is neceſſary to inſert the Perſon or Perſons Names, who

who hold any particular Parcel or Parcels of such Town-Land, as mear the Parts surveyed.

When an Angle is very obtuse, as most in our present Figure are, *viz.* the Angles at A, B, C, E and G; it will be best to lay a Chain from the angular Point as at A, on each of the containing Sides to *c* and to *d*; and any where nearly in the Middle of the Angle as at *e*: measuring the Distances *ce* and *ed*; and these may be placed for the Angle in the Field-Book. Thus,

No.	Sta.	Angle. C. L.	Dist. C. L.
1		1.03 } 1.09 }	17.65

For when an Angle is very obtuse, the Chord Line, as *cd*, will be nearly equal to the Radii A*c*, and A*d*; so if the Arc *ced* be swept, and the Chord Line *cd* be laid on it, it will be difficult to determine exactly, that Point in the Arc where *cd* cuts it: But if the Angle be taken in two Parts as *ce*, and *ed*; such Chords may with Safety be laid on the Arc, and the Angle thence may be truly determined and constructed.

After the same Manner any Piece of Ground may be surveyed by a Two-Pole Chain.

PROB. II.

To take a Survey of a Piece of Ground from any Point within it, from whence all the Angles can be seen; by the Chain only.

Let

Plate VI. Fig. 6.

Let a Mark be fixed at any Point in the Ground, as at H, from whence all the Angles can be ſeen; let the Meaſures of the Lines HA, HB, HC, *&c.* be taken to every Angle of the Field from the Point H; and let thoſe be placed oppoſite to No. 1, 2, 3, 4, *&c.* in the ſecond Column of Radii: The Meaſures of the reſpective Lines of the Mearing, *viz.* AB, BC, CD, DE, *&c.* being placed in the third Column of Diſtances, will compleat the Field-Book. Thus,

Remarks.	No.	Radii. C. L.	Diſtances. C. L.
	1	20.00	17.65
	2	21.72	18.50
	3	21.74	28.00
	4	25.34	20.00
	5	17.20	14.83
	6	29.62	19.41
	7	21.20	24.53
	Cloſe at the firſt Station.		

If any Line of the Field be inacceſſible, as ſuppoſe CD to be, then by Way of Proof that the Diſtance CD is true, let the Meaſure of the Angle CHD be taken by the Line *oo*, with the Chain: If this Angle correſponds with its containing Sides, the Length of the Line DC is truly obtained, and the whole Work is truly taken.

Note, That in ſetting off an Angle it is neceſſary to uſe the largeſt Scale of Equal Parts, *viz.*

that of the Inch, which is diagonally divided into 100 Parts, in order that the Angle ſhould be accurately laid down; or if two Inches were thus divided for Angles, it would be the more exact; for it is by no Means neceſſary that the Angles ſhould be laid from the ſaid Scale with the ſtationary Diſtances.

PROB. III.

To take a Survey by the Chain only, when all the Angles cannot be ſeen from one Point within.

Plate VI. Fig. 7.

Let the Ground to be ſurveyed be repreſented by 1, 2, 3, 4, *&c.* Since all the Angles cannot be ſeen from one Point, let us aſſume 3 Points as A, B, C, from whence they may be ſeen; at each of which let a Mark be put, and the reſpective Sides of the Triangle be meaſured and ſet down in the Field-Book; let the Diſtances from A to 1, and from B to 1, be meaſured, and theſe will determine the Point 1; let the other Lines which flow from A, B, C, as well as the Circuit of the Ground, be then meaſured as the Figure directs; and thence the Map may be eaſily conſtructed.

There are other Methods may be uſed; as dividing the Ground into Triangles, and meaſuring the 3 Sides of each; or by meaſuring the Baſe and Perpendicular of each Triangle. But this we ſhall ſpeak of hereafter.

PROB. IV.

How to take any inaccessible Distance, by the Chain only.

Plate VIII. Fig. 8.

Suppose AB to be the Breadth of a River, or any other inaccessible Distance, which may be required.

Let a Staff or any other Object be set at B. draw yourself backward to any convenient Distance C, so that B may cover A: From B, lay off any other Distance by the River's Side to E, and compleat the Parallelogram EBCD: Stand at D, and cause a Mark to be set at F, in the Direction of A; measure the Distance in Links from E to F, and FB will be also given. Wherefore EF : ED : : FB : AB. Since it's plain (from Part 2. Theo. 3. Sect. 1, and Theo. 2. Sect. 1.) the Triangles EFD, BFA, are mutually equi-angular.

If Part of the Chain be drawn from B to C, and the other Part from B to E; and if the Ends at E and C be kept fast, it will be easy to turn the Chain over to D, so as to compleat a Parallelogram; by reckoning off the same Number of Links you had in BC, from E to D, and pulling each Part streight.

OF THE

CIRCUMFERENTOR.

THIS Inſtrument is compoſed of a Braſs circular Box, about five or ſix Inches in Diameter; within which is a Braſs Ring, divided on the Top into 360 Degrees, and numbered 10, 20, 30, *&c.* to 360: In the Center of the Box is fixed a Steel Pin finely pointed, called a Center-pin, on which is placed a Needle touched by a Loadſtone, which always retains the ſame Situation; that is, it always points to the North and South Points of the Horizon nearly, when the Inſtrument is horizontal, and the Needle at reſt.

The Box is covered with a Glaſs Lid, in a Braſs Rim, to prevent the Needle being diſturbed by Wind or Rain, at the Time of Surveying: There is alſo a Braſs Lid, or Cover, which is laid over the former to preſerve the Glaſs in carrying the Inſtrument.

This Box is fixed, by Screws, to a Braſs Index, or Ruler, of about 14 or 15 Inches in Length, to the Ends whereof are fixed Braſs Sights, which are ſcrewed to the Index, and ſtand perpendicular thereto: In each Sight is a large and a ſmall Aperture, or Slit, one over the other; but theſe are changed, that is, if the large Aperture be uppermoſt

in

in the one Sight it will be loweſt in the other, and ſo of the ſmall ones: Therefore the ſmall Aperture in one is oppoſite to the large one in the other; in the Middle of which laſt, there is placed a Horſe-hair, or fine Silk Thread.

The Inſtrument is then fixed on a Ball and Socket; by the Help of which and a Screw, you can readily fix it horizontally in any given Direction; the Socket being fixed on the Head of a three-legged Staff, whoſe Legs when extended ſupport the Inſtrument, whilſt it is uſed.

How to take Field-Notes by the Circumferentor.

Plate VI. Fig. 6.

Let your Inſtrument be fixed at any Angle as A, your firſt Station; and let a Perſon ſtand at the next Angle B, or cauſe a Staff, with a white Sheet to be ſet there perpendicularly for an Object to take your View to: Then having placed your Inſtrument horizontal (which is eaſily done by turning the Box ſo, that the Ends of the Needle may be equidiſtant from its Bottom, and it traverſes or plays freely) turn the *Flower-de-Luce* or North Part of the Box to your Eye, and looking through the ſmall Aperture, turn the Index about, 'till you cut the Perſon or object in the next Angle B, with the Horſe hair or Thread of the oppoſite Sight: The Degrees then cut by the South End of the Needle, will give the Number to be placed in the ſecond Column of your Field-Book in a Line with Station, No. 1, and expreſſes the Number of Degrees the ſtationary Line is from the North, counting quite round with the Sun.

Moſt Needles are pointed at the South End, and have a ſmall Ring at the North: Such Needles are better than thoſe which are pointed at each End, becauſe the Surveyor cannot miſtake by counting to a wrong End; which Error may be frequently committed, in uſing a two-pointed Needle.

Two-pointed Needles have ſometimes a Ring, but more uſually a Croſs towards the North End; and the South End is generally bearded towards its Extremity, and ſometimes not, but its Arm is a naked right Line from the Cap at the Center.

Having taken the Degrees or Bearing of the firſt ſtationary Line AB, let the Line be meaſured, and the Length thereof in Chains and Links be inſerted in the third Column of your Field-Book, under the Title of Diſtances, oppoſite to Station, No. 1.

It is cuſtomary, and even neceſſary, to cauſe a Sod to be dug up at each Station, or Place where you fix the Inſtrument; to the end, that if any Error ſhould ariſe in the Field-Book, it may be the more readily adjuſted and corrected, by trying over the former Bearings and ſtationary Diſtances.

Having done with your firſt Station, ſet the Inſtrument over the Hole or Spot where your Object ſtood, as at B, for your ſecond Station, and ſend him forward to the next Angle of the Field, as at C; and having placed the Inſtrument in an horizontal Direction, with the Sights directed to the Object at C, and the North of the Box next your Eye, count your Degrees to the South End of the Needle, which regiſter in your Field-Book, in the

ſecond

ſecond Column oppoſite to Station, No. 2; then meaſure the ſtationary Diſtance BC, which inſert in the third Column, and thus proceed from Angle to Angle, ſending your Object before you, 'till you return to the Place where you began, and you will have the Field-Book compleat; obſerving always to ſignify the Parties Names who hold the contiguous Lands, and the Names of the Town-Lands, Rivers, Roads, Bogs, Loughs or Lakes, &c. that mear the Land you ſurvey, as before; and this is the Manner of taking Field Notes by what are called Fore-Sights.

But the Generality of Mearſmen frequently ſet themſelves in diſadvantageous Places, ſo as often to occaſion two or more Stations to be made, where one may do, which creates much Trouble and Loſs of Time: We will therefore ſhew how this may be remedied, by taking of Back-Sights, thus: Let your Object ſtand at the Point where you begin your Survey, as at A; leaving him there, proceed to your next Angle B, where fix your Inſtrument ſo, that you may have the longeſt View poſſible towards C. Having ſet the Inſtrument in an horizontal Poſition, turn the South Part of the Box next your Eye, and having cut your Object at A, reckon the Degrees to the South Point of the Needle, which will be the ſame as if they were taken from the Object to the Inſtrument, the Direction of the Index being the ſame. Let the Degree be inſerted in the Field-Book, and the ſtationary Diſtance be meaſured and annexed thereto, in its proper Column; and thus proceed from Station to Station, leaving your Object in the laſt Point you left, 'till you return to the firſt Station A.

By

By this Method your Stations are laid out to the beſt Advantage, and two Men may do the Buſineſs of three, for one of thoſe who chain may be your Object; but in Fore-Sights, you muſt have an Object before you, beſides two Chainmen.

It was ſaid before, that a Surveyor ſhould have a Perſon with him to carry the hinder End of the Chain, on whom he can depend: This Perſon ſhould be expert and ready at taking Off-ſets, as well as exact in giving a faithful Return of the Length of every ſtationary Line. One who has ſuch a Perſon, and who uſes Back-Sights, will be able to go over near double the Ground he could at the ſame Time, by taking Fore-Sights. But if a Surveyor has no ſuch Perſon on whom he can with Safety depend, he muſt take Fore-Sights, becauſe of overſeeing the Chaining; for ſhould he take Back-Sights, he muſt be obliged, after taking his Degree, to go back to the foregoing Station, to overſee the Chaining, and by this Means to walk three Times over every Line, which is a Slavery not to be borne.

Or a Back and a Fore-Sight may be taken at one Station, thus; with the South of the Box to your Eye, obſerve from B the Object A, and ſet down the Degree in your Field-Book, cut by the South End of the Needle. Again, from B obſerve an Object at C, with the North of the Box to your Eye, and ſet down the Degree cut by the South Point of the Needle, ſo have you the Bearings of the Lines AB and BC; you may then ſet up your Inſtrument at D, from whence take a Back-Sight to C, and a Fore-Sight to E; thus the Bearings may be taken quite round, and the ſtationary Diſtances being annexed to them, will compleat the Field-Book.

But

But in this laſt Method, Care muſt be taken to ſee that the Sights have not the leaſt Caſt on either Side; if they have, it will deſtroy all: And yet with the ſame Sights you may take a Survey by Fore-Sights, or by Back-Sights only, with as great Truth as if the Sights were ever ſo erect, provided the ſame Caſt continues without any Alteration: But upon the whole, Back-Sights only will be found the readieſt Method.

If your Needle be pointed at each End, in taking Fore-Sights, you may turn the North Part of the Box to your Eye, and count your Degrees to the South Part of the Needle, as before; or you may turn the South of the Box to your Eye, and count your Degrees to the North End of the Needle.

But in Back-Sights you may turn the North of the Box to your Eye, and count your Degrees to the North Point of the Needle; or you may turn the South of the Box to your Eye, and count your Degrees to the South End of the Needle.

The Braſs Ring in the Box is divided on the Side into 360 Degrees, thus; from the North to the Eaſt into 90, from the North to the Weſt into 90, from the South to the Eaſt into 90, and from the South to the Weſt into 90 Degrees; ſo the Degrees are numbered from the North to the Eaſt or Weſt, and from the South to the Eaſt or Weſt.

The Manner of uſing this Part of the Inſtrument is this: Having directed your Sights to the Object, whether Fore or Back, as before; obſerve the two Cardinal Points of your Compaſs the Point of the

Needle lies between, (the North, South, Eaſt and Weſt being called the four Cardinal Points, and are graved on the Bottom of the Box,) putting down thoſe Points together by their initial Letters, and thereto annexing the Number of Degrees, counting from the North or South, as before, thus: If the Point of your Needle lies between the North and Eaſt, North and Weſt, South and Eaſt, or South and Weſt Points in the Bottom of the Box, then put down NE, NW, SE, or SW, annexing thereto the Number of Degrees cut by the Needle on the Side of the Ring, counting from the North or South, as before.

But if the Needle points exactly to the North, South, Eaſt, or Weſt, you are then to write down N, S, E, or W, without annexing any Degree.

This is the Manner of taking Field Notes, whereby the Content of Ground may be univerſally determined by Calculation; and they are ſaid to be taken by the Quarter'd Compaſs, or by the four Nineties.

To find the Number of Degrees, contained in any given Angle.

Set up your Inſtrument at the Angular Point, and thence direct the Sights along each Leg of the Angle, and note down their reſpective Bearings as before; the Difference of theſe Bearings, if leſs than 180, will be the Quantity of Degrees contained in the given Angle; but if more, take it from 360, and the Remainder will be the Degrees contained in the given Angle.

OF

OF THE

THEODOLITE.

THIS Inſtrument is a Circle, commonly of Braſs, of ten or twelve Inches in Diameter, whoſe Limb is divided into 360 Degrees, and thoſe again are ſubdivided into ſmaller Parts as the Magnitude of it will admit; ſometimes by equal Diviſions, and ſometimes by Diagonals, drawn from one concentric Circle of the Limb to another.

In the Middle is fixed a Circumferentor, with a Needle; but this is of little or no Uſe, except in finding a Meridian Line, or the proper Situation of the Land.

Over the Braſs Circle is a Pair of Sights, fixed to a moveable Index, which turns on the Center of the Inſtrument, and upon which the Circumferentor Box is placed.

This Inſtrument will either give the Angles of the Field, or the Bearing of every ſtationary Diſtance Line, from the Meridian; as the Circumferentor and Quartered Compaſs do.

First then, *To take the Angles of the Field.*

Plate VI. Fig. 6.

Lay the Ends of your Index to 360, and 180;

turn the whole about with the 360 from you, direct the Sights from A to G, and ſcrew the Inſtrument faſt; direct them from A, to cut the Object at B; the Degree then cut by that End of the Index which is oppoſite to you, will be the Quantity of the Angle GAB, to place in your Field-Book; to which annex the Meaſure of the Line AB, in Chains and Links: Set up your Inſtrument at B, unſcrew it, and lay the Ends of your Index to 360, and 180; turn the whole about with the 360 from you, or 180 next you, 'till you cut the Object at A; ſcrew the Inſtrument faſt, and direct your Sights to the Object at C, and the Degree then cut by that End of the Index which is oppoſite to you, will be the Quantity of the Angle ABC. Thus proceed from Station to Station, ſtill laying the Index to 360, turning it from you, and obſerving the Object at the foregoing Station, ſcrewing the Inſtrument faſt, and obſerving the Object at the following Station, and counting the Degrees to the oppoſite End of the Index, will give you the Quantity of each reſpective Angle.

L E M M A.

All the Angles of any Polygon, are equal to twice as many Right-Angles as there are Sides leſs by four. Thus, all the Angles A, B, C, D, E, F, G, are equal to twice as many Right-Angles, as there are Sides in the Figure, leſs by four.

Plate VI. Fig. 6.

Let the Polygon be diſpoſed into Triangles, by Lines drawn from any aſſigned Point H within it, as by the Lines HA, HB, HC, *&c.* It is evident then

then (by Theo 5. Sect 1.) that the three Angles of each Triangle are equal to two right; and consequently, that the Angles in all the Triangles, are twice as many right ones as there are Sides: But all the Angles about the Point H, are equal to four right (by Cor. 2. Theo. 1. Sect. 1.); therefore the remaining Angles are equal to twice as many right ones as there are Sides in the Figure, abating four. Q. E. D.

SCHOLIUM.

Hence we may know if the Angles of a Survey be truly taken: For if their Sum be equal to twice as many Right-Angles, as there are Stations, abating four Right-Angles, you may conclude that the Angles were truly taken, otherwise not.

If you take the Bearing of any Line with the Circumferentor, that Bearing will be the Number of Degrees the Line is from the North; consequently the North must be a like Number of Degrees from the Line, and thus the North, and of course the South, as well as the East and West, or the Situation of the Land, is obtained.

Secondly, *To take the Bearing of each respective Line from the Meridian; or to perform the Office of the Circumferentor, or Quarter'd Compass, by the Theodolite.*

Set your Instrument at the first Station, and lay the Index to 360 and 180, with the *Flower-de-Luce* of the Box next the 360; unscrew the Instrument, and turn the whole about, 'till the North and South Points of the Needle cut the North and South Points in the Box; then screw it fast, so is the Instrument

North,

North, and South, abstracted of the Variation.

The Circumferentor Box may be then taken off.

Direct the Sights to the Object at the second Station, and the Degree cut by the opposite End of the Index will be the Bearing of that Line from the North, and the same that the Circumferentor would give.

After having measured the stationary Distance, set up your Instrument at the second Station; unscrew it, and set either End of the Index to the Degree of the last Line, and turning the whole about with that Degree towards you, direct your Sights to an Object at the foregoing Station, and screw the Instrument fast; it will then be parallel to its former Situation, and consequently North and South: Direct then your Sights to an Object at the following Station, and the Degree cut by the opposite End of the Index, will be the Bearing of that Line.

In like Manner you may proceed through the whole.

If the Brass Circle be divided into four Nineties, from 360 and 180, and the Letters N, S, E, W, be applied to them; the Bearings may be obtained by putting down the Letters the far or opposite End of the Index lies between, and annexing thereto the Degrees from the N or S; and this is the same as the Quarter'd Compass.

If you keep the Compass Box on, to see the mutual Agreement of the two Instruments; after having fixed the Theodolite North and South, as before,

before; turn the Index about with the North End, or *Flower-de-Luce*, next your Eye, and count the Degree to the opposite, or South End of the Index, and this will correspond with the Degree cut by the South End of the Needle.

At the Second, or next Station, unscrew the Instrument, and set the South of the Index to the Degree of the last Station; turn the whole about, with the South of the Index to you, and cut the Object at the foregoing Station; screw the Instrument fast, and with the North of the Index to you, cut the Object at the next following Station, the Degree then cut by the South of the Index will correspond with the Degree cut by the South End of the Needle, and so through the whole.

Some Theodolites have a standing Pair of Sights fixed at 360 and 180, besides those on the moveable Index: If you would use both, look thro' the standing Sights, with the 180 next you, to an Object at the foregoing Station; screw the Instrument fast, and direct the upper Sights on the moveable Index, to the Object at the following Station, and the Degree cut by the opposite End of the Index, will give you the Quantity of the Angle of the Field.

Two Pair of Sights can be of no Use in finding the Angles from the Meridian; and inasmuch as one Pair is sufficient to find the Angles of the Field, the second can be of no Use: Besides, they obstruct the free Motion of the moveable Index, and therefore are rather an Incumbrance than of any real Use. Some will have it, that they are useful with the others, for setting off a right Angle, in taking an Off-set; and surely this is as easily performed by

the one Pair on the moveable Index: Thus, if you lay the Index to 360, and 180, and cut the Object either in the last or following Station, screw the Instrument fast, and turn the Index to 90 and 270, and then it will be at Right Angles with the Line. So that the small Sights, or those to the Circle, can be of no additional Use to the Instrument, and therefore should be laid aside as useless.

This Instrument may be used in windy and rainy Weather, as well as in mountainous and hilly Grounds; for it does not require an horizontal Position to find the Bearing, or Angle, as the Needle doth; and therefore is preferable to any Instrument that is governed by the Needle.

Mr. *Gabriel Stokes*, formerly Mathematical Instrument Maker, and Surveyor of Land, in *Essex-Street Dublin*. Has contrived a small Instrument of this Nature, about six or seven Inches Diameter, which is accurately divided, and is much more portable than a Theodolite, which he calls a *Pantometron*: Which may be had uppon application to Mr. *Thos. Reading*, Mathematical Instrument maker and Surveyor of Land, who served his time to the said Mr. *Stokes*, at present dwelling in *Georges Lane*, near *Stephens Street*, *Dublin*.

OF

Of the SEMICIRCLE.

THIS Inſtrument, as its Name imports, is a half Circle, divided from its Diameter into 180 Degrees, and from thence again, that is, from 0, to 360: It is generally made of Braſs, and is from 8 to 16 Inches Diameter.

On the Center there is a moveable Index with Sights, on which is placed a Circumferentor-Box, as in the Theodolite.

This Inſtrument may be uſed as the Theodolite in all Reſpects; but with this Difference, when you are to reckon the Degree to that End of the Index which is off of the Semicircle, you may find it at the other End, reckoning the Degree from 180 forwards.

Of the PLANE TABLE.

A PLANE TABLE is an *Oblong* of Oak, or other Wood, about 15 Inches long, and 12 broad; they are generally composed of 3 Boards, which are easily taken asunder, or put together, for the Conveniency of Carriage.

There is a Box Frame, with 6 Joints in it, to take off and put on as Occasion serves; it keeps the Table together, and is likewise of Use to keep down a Sheet of Paper which is put thereon.

The Outside of the Frame is divided into Inches and Tenths, which serve for ruling Parallels or Squares on the Paper, or for shifting it, when Occasion serves.

The Inside of the Frame is divided into 360 Degrees, which, tho' unequal on it, yet are the Degrees of a Circle produced from its Center, or Center of the Table, where there is a small Hole.

The Degrees are subdivided as small as their Distance will admit; at every tenth Degree are two Numbers, one the Number of Degrees, the other its Complement to 360.

There is another Center Hole, about $\frac{1}{4}$ of the Table's Breadth from one Edge, and is in the Middle between the two Ends. To this Center Hole on the other Side of the Frame, there are the Divisions of a Semicircle, or 180 Degrees; and these again are subdivided into Halfs, or Quarters, as the Size of the Instrument will admit.

That

That Side of the Frame on which the 360 Degrees are, supplies the Place of a Theodolite, the other that of a Semicircle.

There is a Circumferentor Box of Wood, with a Paper Chart at the Bottom, applied to one Side of the Table by a Dove-tail Joint, fastened by a Screw. This Box (besides its rendering the Plane Table capable of answering the End of a Circumferentor) is very useful for placing the Instrument in the same Position every Remove.

There is a Brass Ruler or Index, of about two Inches broad, with a sharp or fiducial Edge, at each End of which is a Sight: On the Ruler are Scales of equal Parts, with and without Diagonals, and a Scale of Chords; the whole is fixed on a Ball and Socket, and set on the three legged Staff.

1. *To take the Angles of the Field by the Table.*

Having placed the Instrument at the first Station, turn it about 'till the North End of the Needle be over the Meridian, or *Flower-de-Luce* of the Box, and there screw it fast. Assign any convenient Point, to which apply the Edge of the Index, so as thro' the Sights you may see the Object in the last Station, and by the Edge of the Index from the Point draw a Line. Again, turn about the Index with its Edge to the same Point, and thro' the Sights observe the Object in the second Station, and from the Point, by the Edge of the Index, draw another Line; so is the Angle laid down: On that last Line set off the Distance to the second Station, in Chains and Links: Apply your Instrument to the second Station, taking the Angle as before; and after the like Manner proceed 'till the whole is finished.

This Method may be used in good Weather, if the Needle be well touched and plays freely; but if it be in windy Weather, or the Needle out of Order, it is better, after having taken the first Angle as before, and having removed your Instrument to the second Station, and placed the Needle over the Meridian Line as before, to lay the Index on the last drawn Line, and look backward thro' the Sights; if you then see the Object in the first Station, the Table is fixed right, and the Needle is true; if not, turn the Table about, the Index lying on the last Line, 'till thro' the Sights you see the Object in the first Station; and then screw it fast, and keeping the Edge of the Index to the second Station, direct your Sights to the next; draw a Line by the Edge of the Index, and lay off the next Line; and proceed so thro' the whole without using the Needle, as you do with the *Theodolite*.

If the Sheet of Paper on the Table be not large enough to contain the Map of the Ground you survey, you must put on a clean Sheet, when the other is full; and this is called Shifting of Paper, and is thus performed.

Plate VI. Fig. 8.

Let ABCD represent the Sheet of Paper on the Plane Table, upon which the Plot E, F, G, H, I, K, L, M, is to be drawn; let the first Station be E, proceed as before from thence to F, and to G; then proceeding to H, you find there is not Room on your Paper for the Line GH: However, draw as much of the Line GH, as the Paper can hold, or draw it to the Paper's Edge. Move your Instrument back to the first Station E, and proceed the contrary Way to M, and to L; but in going from

from thence to K, you again find your Sheet won't hold it; however, draw as much of the Line LK on the Sheet, as it can hold.

Take that Sheet off the Table, first observing the Distance *oo* of the Lines GH, and LK, by the Edge of the Table; take off that Sheet, and mark it with No. 1, to signify it to be the first taken off. Having then put on another Sheet, lay that Distance *oo* on the contrary End of the Table, and so proceed as before with the Residue of the Survey, from *o* to H, to K, and thence to *o*; so is your Survey compleat.

In the like Manner you may proceed to take off, and put on, as many Sheets as are convenient; and these may afterwards be joined together with Mouth Glue, or fine white Water very thin.

If the Index be fixed to the first Center, using the 360 Side, it will then serve as a Theodolite, and when to the second Center, using the 180 Side, it will serve as a Semicircle; by either of which you may survey in rainy Weather, when you can't have Paper on the Table.

To take an Angle of Altitude by the *Circumferentor*, *Theodolite*, *Semicircle*, or *Plane Table*.

1. *To take an Angle of Altitude, by the Circumferentor.*

LET the Glaſs Lid be taken off, and let the Inſtrument be turned on one Side, with the Stem of the Ball into the Notch of the Socket, ſo that the Circle may be perpendicular to the Plane of the Horizon; let the Inſtrument be placed in this Situation before the Object, ſo that the Top thereof may be ſeen thro' the Sights: Let a Plummet be ſuſpended from the Center Pin, and the Object being then obſerved, the Complement of the Number of Degrees, comprehended between the Thread of the Plummet, and that Part of the Inſtrument which is next your Eye, will give the Angle of Altitude required.

2. If an Angle of Altitude is to be taken by the Theodolite, or Semicircle, let a Thread be run thro' a Hole at the Center, and a Plummet be ſuſpended by it; turn the Inſtrument on one Side, by the Help of the Ball and Notch in the Socket for that Purpoſe, ſo that the Thread may cut 90, having 360 Degrees next you: Screw it faſt in that Poſition, and thro' the Sights cut the Top of the Objects;

Objects; and the Degrees then cut by the End of the Index next you, are the Degrees of Elevation required. An Angle of Depression is taken the contrary Way.

3 By the Plane Table an Angle of Altitude is taken in the like Manner, by suspending a Plummet from the Center thereof, having turned the Table on one Side, and fix'd the Index to the Center by a Screw, so as to move freely, let the Thread cut 90, look thro' the Sights as before, and you have the Angle of Elevation, and on the contrary that of Depression.

OF

OF THE

PROTRACTOR.

THE Protractor is a Semicircle annex'd to a Scale, and is made of Brass, Ivory, or Horn; its Diameter is generally about five or six Inches.

The Semicircle contains three concentric Semicircles, at such Distances from each other, that the Spaces between them may contain Figures.

The outward Circle is numbered from the Right to the left Hand, with 10, 20, 30, *&c.* to 180 Degrees; the middlemost the same Way, from 180, to 360 Degrees; and the innermost, from the upper Edge of the Scale both ways, from 10, 20, 30, *&c.* to 90 Degrees.

It is easy to conceive that the Protractor, tho' a Semicircle, may be made to supply the Place of a whole Circle; for if a Line be drawn, and the Center-hole of the Protractor be laid on any Point in that Line, the upper Edge of the Scale corresponding with that Line, the Divisions on the Edge of the Semicircle will run from 0, to 180, from Right to Left: Again, if it be turned the other Way, or downwards, keeping the Center-hole thereof on the aforesaid Point in the Line, then the Divisions will run from 180 to 360, and

and ſo compleats an entire Circle with the former Semicircle.

The Uſe of the Protractor is to lay off Angles, and to delineate or draw a Map, or Plan, of any Ground from the Field-Notes; and is performed in the following Manner.

To protract a Field-Book, when the Angles are taken from the Meridian.

Plate VI. Fig. 9.

On your Paper, rule Lines parallel to each other, at an Inch aſunder, (being moſt uſual) or at any other convenient Diſtance; on the left End of the Parallels put N, for North, and on the right S, for South; put E at the Top for Eaſt, and W at the Bottom of your Paper for Weſt.

Then let the following Field Book be that which is to be protracted, the Bearings being taken from the Meridian, whether by a Circumferentor, Theodolite, or Semicircle, and meaſured with a Two-Pole Chain.

No.	Bearing.	C. L.
1	$283\frac{1}{2}$	55.20
2	$348\frac{3}{4}$	12.36
3	317	29.20
4	266	55.20
5	193	40.00
6	124	76.00
7	$63\frac{3}{4}$	87.02

Cloſe at the firſt Station.

Pitch upon any convenient Point on your Paper, for your first Station, as at 1, on which lay the Center-hole of your Protractor, with a Protracting-Pin; then if the Degrees be less than 180, turn the Arc of your Protractor downwards, or towards the West; but if more than 180, upwards, or towards the East.

Or if the Right-Hand be made the North, and the Left the South, the West will be then up, and the East down.

In this Case, if the Degrees be less than 180, turn the Arc of your Protractor upwards, or towards the West; and if more, downwards, or to the East.

By the foregoing Field-Book, the first Bearing is $283\frac{1}{2}$; turn the Arc of your Protractor upwards, keeping the Pin in the Center-hole, move the Protractor so that the Parallel Lines may cut opposite Divisions, either on the Ends of the Scale, or on the Degrees, and then it is parallel. This must be always first done, before you lay off your Degrees.

Then by the Edge of the Semicircle keeping the Protractor steady, with the Pin prick the first Bearing $283\frac{1}{2}$, and from the Center Point, thro' that Point or Prick, draw a blank Line with the Pin, on which from a Scale of equal Parts, or from the Scale's Edge of the Protractor, lay off the Distance 55C. 20L. so is that Station protracted.

At

At the End of the first Station, or at 2, which is the Beginning of the second, with the Pin place the Center of the Protractor, turning the Arc up, because the Bearing of the second Station is more than 180, *viz.* 348¾. Place your Protractor parallel as before, and by the Edge of the Semicircle, with the Pin prick at that Degree. thro' which and the End of the foregoing Station, draw a blank Line, and on it set the Distance of that Station.

In the like Manner proceed thro' the whole, only observe to turn the Arc of your Protractor down, when the Degrees are less than 180.

If you lay off the stationary Distances by the Edge of the Protractor, it is necessary to observe, that if your Map is to be laid down by a Scale of 40 Perches to an Inch, every Division on the Protractor's Edge will be one Two-Pole Chain; ½ a Division will be 25 Links, and ¼ of a Division will be 12½ Links.

If your Map is to be laid down by a Scale of 20 Perches to an Inch, two Divisions will be one Two-Pole Chain; one Division will be 25 Links; ½ a Division 12½ Links, and ¼ of a Division will be 6¼ Links.

In the general, if 25 Links be multiplied by the Number of Perches to an Inch, the Map is to be laid down by, and the Product be divided by 20; (or which is the same Thing, if you cut off one and take the half,) you will have the Value of one Division on on the Protractor's Edge, in Links and Parts.

EXAMPLE.

1. How many Links in a Diviſion, if a Map be laid down by a Scale of 8 Perches to an Inch?

$$
\begin{array}{r}
25 \\
8 \\
\hline
2|0\,)\,20|0 \\
\hline
10
\end{array}
$$
Links. Anſwer.

2. How many Links in a Diviſion, if a Map be laid down by a Scale of 10 Perches to an Inch?

$$
\begin{array}{r}
25 \\
10 \\
\hline
2|0\,)\,25|0 \\
\hline
\end{array}
$$

12.5 or $12\frac{1}{2}$ Links. Anſwer.

And ſo of any other.

To protract a Field-Book, taken by the Angles of the Field.

Note, We here ſuppoſe the Land ſurveyed is kept on the right Hand as you ſurvey.

Draw a blank Line with a Ruler of a Length greater than the Diameter of the Protractor; pitch upon any convenient Point therein, to which apply the Center-hole of your Protractor with your Pin, turning the Arc upwards if the Angle be leſs than 180, and downwards if more; and obſerve to keep the upper Edge of the Scale, or 180 and 0 Degrees upon

upon the Line: Then prick off the Number of Degrees contained in the given Angle, and draw a Line from the firſt Point through the Point at the Degrees; upon which lay the ſtationary Diſtance. Let this Line be lengthened forwards and backwards, keeping your firſt Station to the Right, and ſecond to the Left; and lay the Center of your Protractor over the ſecond Station, with your Pin, turning the Arc upwards, if the Angle be leſs than 180, and downwards, if more; and keeping the 180 and 0 Degrees on the Line, prick off the Number of Degrees contained in the given Angle, and thro' that Point and the laſt Station draw a Line, on which lay the ſtationary Diſtance: And in like Manner proceed through the whole.

In all Protractions, if the End of the laſt Station falls exactly in the Point you begin at, the Field-Work and Protraction are truly taken, and performed; if not, an Error muſt have been committed in one of them: In ſuch Caſe, make a ſecond Protraction; if this agrees with the former, and neither meet or cloſe, the Fault is in the Field-Work, and not in the Protraction: And then a Re-ſurvey muſt be taken.

SECT. IV.

Containing Five various Methods by which the Areas of Right-lined Figures may be determined, two of which were never yet published.

DEFINITION.

THE Area or Content of any Plane Surface in Perches, is the Number of Square Perches, that Surface contains.

Plate VII. Fig. 1.

Let ABCD represent a Rectangular Parallelogram, or Oblong: Let the Side AB, or DC, contain 8 equal Parts; and the Side AD, or BC, three of such Parts: Let the Line AB be moved in the Direction of AD, 'till it has come to EF; where AE, or BF, (the Distance of it from its first Situation) may be equal to one of the equal Parts. Here 'tis evident, that the generated Oblong ABEF, will contain as many Squares as the Side AB contains equal Parts, which are 8; each Square having for its Side one of the equal Parts, into which AE, or AD, is divided. Again, let AB move on 'till it comes to GH, so as GE, or HF, may be equal to AE, or BF; then it is plain that the Oblong AGHB, will contain twice as many

Squares

Squares as the Side AB contains Equal Parts. After the ſame Manner it will appear, that the Oblong ADCB will contain three times as many Squares as the Side AB contains equal Parts; and in general that every Rectangular Parallelogram, whether Square or Oblong, contains as many Squares as the Product of the Number of equal Parts in the Baſe, multiplied into the Number of the ſame equal Parts in the Height, contains Units, each Square having for its Side one of the equal Parts.

Hence ariſes the Solution of the following Problems.

PROB. I.

To find the Content of a Square Piece of Ground.

1. Multiply the Baſe in Perches, into the Perpendicular in Perches, (or ſquare the Baſe) the Product will be the Content in Perches; and becauſe 160 Perches make an Acre, it muſt thence follow, that

Any Area, or Content in Perches, being divided by 160, will give the Content in Acres; the remaining Perches, if more than 40, being divided by 40, will give the Roods, and the laſt Remainder, if any, will be Perches.

Or thus:

2. Square the Side in Four-Pole Chains and Links, and the Product will be ſquare Four-Pole Chains and Links; divide this by 10, or cut off one more than the Decimals, which are five in all, from the Right towards the Left: The Figures

reſting

resting to the Left are Acres, because 10 square Four-Pole Chains make an Acre, and the remaining Figures are Decimal Parts of an Acre. Multiply the five Figures to the Right by 4, cutting 5 Figures from the Product, and if any Figure be to the Left of them, it is a Rood, or Roods; multiply the last cut off Figures by 40, cutting off five, or (which is the same thing) by 4, cutting off four; and the remaining Figures to the Left, if any, are Perches.

1. The first Part is plain, from considering that a Piece of Ground in a square Form, whose Side is a Perch, must contain a Perch of Ground; and that 40 such Perches make a Rood, or Stang, and four Roods an Acre; or which is the same Thing, that 160 square Perches make an Acre, as before.

2. A square Four-Pole Chain (that is a Piece of Ground four Poles or Perches every Way) must contain 16 square Perches; and since 160 Perches make an Acre, therefore 10 times 16 Perches, or 10 square Four-Pole Chains make an Acre.

Note, that the Chains given, or required, in any of the following Problems, are supposed Two-Pole Chains, that Chain being most commonly used in this Kingdom.

EXAMPLES.

C. L.

Let ABCD be a square Field, whose Side is 14.29: I demand the Content in Acres.

By

C. L.
By Problem 4. Section 3. 14.29, are equal to 29.16 Perches.

```
  29.16
  29.16
  -----
  17496
  2916
 26244
 5832
 -------       A.  R.  P.
160)850 3056(5. 1. 10. Content.
 ----
40)50( 1 Rood
 ----
 10 Perches.
```

Or thus:

```
C. L.              C. L.
14.29 is equal to 7. 29 of Four-Pole Chains, by
Prob. 1. Sect. 3.  7. 29
                  -----
                  6561
                  1458                A. R. P.
                 5103  Cont. as before 5. 1. 10.
                 ------
          Acres 5|31441
                      4
                 ------
          Rood 1|25764
                      4
                 ------
       Perches 10|3056
```

It is required to lay down a Map of this Piece of Ground, by a Scale of twenty Perches to an Inch.

Take 29.16 the Perches of the given Side, from the ſmall Diagonal on the common Surveying Scale, where 20 ſmall, or two of the large Diviſions are an Inch; make a Square whoſe Side is that Length, (by Prob. 9. Sect. 1.) and it is done.

PROB. II.

To find the Side of a Square, whoſe Content is given.

Extract the Square Root of the given Content in Perches, and you have the Side in Perches, and conſequently in Chains.

EXAMPLE.

It is required to lay out a ſquare Piece of Ground which ſhall contain 12A. 3R. 16P. Required the Number of Chains in each Side of the Square; and to lay down a Map of it, by a Scale of 40 Perches to an Inch.

A. R. P.
12. 3. 16
4

51
40

C. L.
2056(45.34 Perch, which is 22. 33½, by Prob. [6. Sect. 3.

85)456

903)3100

9064)39100

To

To draw the Map.

From a Scale where 4 of the large, or 40 of the ſmall Diviſions are an Inch, take 45·34, the Perches of the Side, of which make a Square.

PROB. III.

To find the Content of an Oblong Piece of Ground.

Multiply the Length by the Breadth, for the Content.

EXAMPLE.

Plate I. Fig. 3.

Let ABCD be an oblong Piece of Ground, whoſe Length AB is 14C. 25L, and Breadth 8C. 37L. I demand the Content in Acres, and alſo to lay down a Map of it, by a Scale of 20 Perches to an Inch.

$$\begin{array}{l} \text{C. L.}\quad\text{Perches.} \\ \left.\begin{array}{r} 14.25=29.00 \\ 8.37=17.48 \end{array}\right\} \text{By Prob. 4. Sect. 3.} \\ \hline \quad 15732 \\ \quad 3496 \\ \hline \qquad\qquad\qquad \text{A. R. P.} \\ 160)506\cdot9200(3.\ 0.\ 27.\ \text{Content.} \\ \quad 26 \text{ Perch, or near } 27. \end{array}$$

Or thus:

	4 Pole C.	
C. L.	C. L.	
14.25 =	7.25	By Prob. 1. Sect. 3.
8.37 =	4.37	
	5075	
	2175	
	2900	
Acres	3|16825	
	4	
Rood	|67300	
	4	
Perches	26|9200	

To draw the Map.

Make an Oblong (by Schol. to Prob. 9. Sect. 1.) whose Length, from a Scale of 20 to an Inch, may be 29 Perches, and Breadth, 17.48 Perches.

PROB. IV.

The Content of an Oblong Piece of Ground, and one Side given, to find the other.

Divide the Content in Perches, by the given Side in Perches, the Quotient is the required Side in Perches; and thence it may be easily reduced to Chains.

EXAMPLE.

EXAMPLE.

C. L.

There is a Ditch 14. 25 long, by the Side of which it is required to lay out an Oblong Piece of Ground, which ſhall contain 3A. 0R. 27P: What Breadth muſt be layed off at each End of the Ditch, to incloſe the 3A. 0R. 27P?

```
A. R. P.
3. 0. 27.
4
———
12
40
——— Perch.   C.  L.
29)507(17.48 = 8. 37 Breadth.
———
217
———
 140
———
 240
———
   8
———
```

The Map is done as the laſt.

PROB. V.

To find the Content of a Piece of Ground, in Form of an Oblique Angular Parallelogram; or of a Rhombus, or Rhomboides.

Multiply the Baſe into the Perpendicular Height. The Reaſon is plain from Theo. 13. Sect. 1.

EXAMPLE.

Example.

Plate VII. Fig. 2.

Let ABCD be a Piece of Ground in Form of a Rhombus, whose Base AB is 22 Chains, and Perpendicular DE, or FC, 20 Chains. Required the Content.

$$\begin{array}{l} \text{C.} \quad \text{C.} \\ \left.\begin{array}{l} 22=11.0 \\ 20=10.0 \end{array}\right\} \text{ 4 Pole Chains.} \\ \hline \text{Acres } 11|0 \end{array}$$

Or,

$$\begin{array}{l} \text{C.} \\ \left.\begin{array}{l} 22=44 \\ 20=40 \end{array}\right\} \text{ Perches.} \\ \hline 160)1760(11 \text{ Acres.} \\ \quad\quad 160 \\ \hline \end{array}$$

The Converse of this is done by Prob. 4. and the Map is drawn, by laying of the Perpendicular on that Part of the Base from whence it was taken: Joining the Extremity thereof to that of the Base, by a Right Line, and thence compleat the Parallelogram.

PROB.

PROB. VI.

To find the Content of a Triangular Piece of Ground.

Multiply the Base by Half the Perpendicular, or the Perpendicular by Half the Base; or take Half the Product of the Base into the Perpendicular.

The Reason hereof is plain, from Cor. 2. Theo. 12. Sect. 1.

EXAMPLE.

Plate I. Fig. 16.

Let ABC be a Triangular Piece of Ground, whose longest Side or Base BC, is 24 C. 38 L. and Perpendicular AD, let fall from the opposite Angle, is 13 C. 28 L. Required the Content.

C. L. C. L.
1. Base 24. 38 = 12. 38 } 4 Pole Chains.
$\frac{1}{2}$ Perp. 3. 39

11142
3714
3714

Acres 4|19682
4

Rood 7|8728
40

Perches 31)4912

A. R. P.
Content 4. 0. 31.

	C. L.	C. L.	
Perp.	13.28	= 6.78	Four-Pole Chains, by Prob.
½ Perp.	6.39	= 3.39	1. Sect. 3.

Or 2dly. Perp. 6.78 of Four-Pole Chains.
½ Base 6.19

```
     6102
      678
   4068
   ------  A.  R.  P.
  4|19682 = 4.  0.  31.
```

Or 3dly. Base 12.38 Four-Pole Chains.
Perp. 6.78

```
      9904
     8666
    7428
    ------
    839364
    ------  A.  R.  P.
Its ½ 4|19682 = 4.  0.  31.
```

Or the Base, and Perpendicular, may be reduced to Perches; and the Content may be thence obtained thus;

1. Base

	C. L.	Perches.	
Perp.	13.28	=27.12	By Prob. 4. Sect. 3.
Half the Perp.		13.56	

		Perches.	C. L.
1.	Base	49.52	=24.38
	½ Perp.	13.56	

```
    29712
   24760
  14856
  4952
  ---------- A. R. P.
160)671.4912(4. 0. 31.
  -----
   31
```

		Perch.
2.	Perp.	27.12
	Half Base	24.76

```
    16272
   18984
  10848
  5424
  ---------- A. R. P.
  671.4912=4. 0. 31.
```

		Perch.
3	Base	49.52
	Perp.	27.12

```
          9904
         4952
       34664
       9904
     ─────────
     1342.9824
     ─────────   A. R. P.
      671.4912=4. 0. 31.
```

The Map may be readily drawn, having the Distance from either End of the Base, to the Perpendicular given; as may be evident from the Figure.

PROB. VII.

The Content of a Triangular Piece of Ground, and the Base given, to find the Perpendicular.

Divide the Content in Perches, by half the Base in Perches; and the Quotient will give you the Perpendicular in Perches, and so in Chains.

Examples.

Plate I. Fig. 16.

Let BC be a Ditch, whose Length is 24C. 40L. by which it is required to lay out a Triangular Piece of Ground, whose Content shall be 4A. 1R. 10P. Required the Perpendicular.

Base

C. L. Perch.
Baſe 24.40=49.6
Half the Baſe=24.8

A. R. P.
4. 1. 10.
4
———
17
40
——— Perches.
24.8)690(27.82
———
1940
———
2040
———
560
———
64
———

Perch. C. L.
Anſwer Perp. 27.82=13.45

This Perpendicular being laid on any Part of the Baſe, and Lines run from its Extremity to the Ends of the Baſe, will lay out the Triangle (by Cor. to Theo. 13. Sect. 1.) ſo that the Perpendicular may be ſet on that Part of the Baſe, which is moſt convenient and agreeable to the Parties concerned.

LEMMA.

If from half the Sum of the Sides of any Plane Triangle ABC, each particular Side be taken; and if the half Sum, and the three Remainders be multiplied continually into each other, the Square Root of this Product will be the Area of the Triangle.

Plate VIII. Fig. 9.

Bisect any two of the Angles, as A and B, with the Lines AD, BD, meeting in D; draw the Perpendiculars DE, DF, DG.

The Triangle AFD is equiangular to AED; for the Angle FAD=EAD by Construction, and AFD=AED, being each a right Angle, and of Consequence ADF=ADE; wherefore AD : DF :: AD : DE : And since AD bears the same Proportion to DF, that it doth to DE, DF=DE, and the Triangle AFD=AED. The same way DE=DG, and the Triangle DEB=DGB, and FD=DE =DG; therefore D will be the Center of a Circle that will pass through E, F, G.

In the same Way if A and C were bisected, the same Point D would be had; therefore a Line from D to C will bisect C, and thus the Triangles DFC DGC will be also equal.

Produce CA to H, 'till AH=EB or GB; so will HC be equal to half the Sum of the Sides, *viz.* to $\frac{1}{2}$ AB, + $\frac{1}{2}$ AC + $\frac{1}{2}$ BC: For FC, FA, EB, are severally equal to CG, AE, BG; and all these together are equal to the Sum of the Sides of the Triangle; therefore FC + FA + EB or CH, are equal to half the Sum of the Sides.

FC

FC = CH—AB, for AF = AE, and HA—EB; therefore HF = AB: And AF = CH—BC; for CF = CG, and AH = GB; therefore BC = HA+FC, and AH = CH—AC.

Continue DC, 'till it meets a Perpendicular drawn upon H, in K; and from K draw the Perpendicular KI, and join AK.

Because the Angles AHK and AIK are two right ones, the Angles HAI and K together, are equal to two right; since the Angles of the two Triangles contain four right: In the same Way FDE+FAE = (2 right Angles =) FAE + IAH; let FAE be taken from both, then FDE = IAH, and of course FAE = K: The quadrilateral Figures AFDE, and KHAI, are therefore similar, and have the Sides about the equal Angles proportional; and it is plain the Triangles CFD and CHK are also proportional: Hence,

FD : HA : : FA : HK
FD : FC : : HK : HC

Wherefore by multiplying the Extreams, and Means in both, it will be, the Square of FDxHK x HC = FC x FA x HA x HK; let KH be taken from both, and multiply each Side by CH; then the Square of CH x by the Square of FD = FC x FA x HA x CH.

It is plain, by the foregoing Problem, that $\frac{1}{2}$ AB x DE, + $\frac{1}{2}$ BC x DG + $\frac{1}{2}$ AC x FD = the Area of the Triangle; or that Half the Sum of the Sides, *viz.* CH x FD = the Triangle; wherefore the Square of CH x by the Square of FD = FC x

FA

FA x HA x CH, that is, the half Sum multiplied continually into the Differences between the half Sum and each Side, will be the Square of the Area of the Triangle, and its Root the Area. Q. E. D.

Hence the following Problem will be evident.

P R O B. VIII.

The Sides of any plain Triangle given to find the Area.

Plate VIII. Fig. 9.

Add the Sides together, and from the half Sum take each Side ſeverally: Multiply the half Sum and three Remainders continually into each other, and the Square-Root of their Product will be the Area.

If the Triangle be meaſured by a Two-Pole Chain, reduce the Sides to Four-Pole Chains; and let them ſtand thus:

		C. L.	
	AB	10. 64	
Given	AC	12. 28	the Area required.
	BC	9. 00	
	Sum	31. 92	
	Half Sum	15. 96	

15.96

C. L. C. L.
15.96—10.64=5.32 first Remainder.
15.96—12.28=3.68 second Remainder.
15.96— 9.00=6.96 third Remainder.

C. L.
15.96 half the Sum.
5.32 first Remainder.

3192
4788
7980

84.9072
3.68 second Remainder.

6792576
5094432
2547216

312.458496
6.96 third Remainder.

1874750976
2812126464
1874750976

2174.71013216

```
        2174.710132 16(46 6337
        16
        ----
    86 | 574
       | 516
       ------
   926 | 5871
       | 5556
       -------
  9323 | 31501
       | 27969
       -------
 93263 | 353232
       | 279789
       -------
932667 | 7344316
       | 6528669
       --------
          815647
       --------
```

Area in square Four-Pole Chains,	46.6337
Acres	4.66337
	4
Roods	2.65348
	40
Perches	26.13920

	A.	R.	P.
Anſwer	4.	2.	26.

Or if you reduce each Side into Perches, the Operation is performed in the like Manner; and the Square Root will be Perches.

To

To perform this by Logarithms.

Inasmuch as the Addition of Logarithms answers the Multiplication of their corresponding Numbers; and that the Number answering to the half of a Logarithm, will give the Square Root of the Number of that Logarithm, it follows,

That half the Sum of the Logarithms of half the Sum of the Sides, and of the three Remainders, will give the Area. Thus,

Half Sum	15.96	1.20303
First Remainder	5.32	0.72591
2d Remainder	3.68	0 56585
3d Remainder	6.96	0.84261
		3.33740
Square 4 Pole Chains	46.63	1.66870
Acres	4.663	
	4	
Roods	2.652	
	4	
Perches	26.08	

	A.	R.	P.
Answer, as before	4.	2.	26.

If therefore you reduce a Piece of Ground into Triangles, (as that in Plate VII. Fig. 4.) and measure the three Sides of each; you may thus find the Content of the whole, by natural Numbers, or more readily by Logarithms.

The Figure may alſo be eaſily conſtructed by Prob. 1. Sect. 1.

PROB. IX.

To find the Content of a Trapezium Piece of Ground.

Multiply the Diagonal, or Baſe, by half the Sum of the two Perpendiculars, or the two Perpendiculars by half the Baſe; or take half a Product of the Baſe into the Sum of the two Perpendiculars.

DEFINITION.

A Line contained between the two remoteſt Angles, is the Diagonal.

EXAMPLE.

Plate VII. Fig. 3.

Let ABCD be a Piece of Ground, in Form of a Trapezium, whoſe Diagonal, or Baſe, AC, is 32C. 10L; its Perpendicular bB on one Side of that Line is 6 C. 41 L. and Perpendicular dD on the other Side is 13 C. 31 L. Required the Content.

Perp

	C. L.	C. L.	
Perp.	6.41 =	3.41	4 Pole Chains.
Perp.	13.31 =	6.81	
Sum		10.22	
Half Sum		5.11	

```
      C. L.    C. L.
Base  32.10 = 16.10
               5.11
             ------
               1610
              1610
             8050
             --------
Acres        8|22710
                   4
             --------
Roods         |90840
                   4
             --------
Perches      36|336
```

The other two Methods are obvious, from what has been laid down (in the Triangle, Prob. 6.) and will produce a like Content. If both the Base and the Sum of the two Perpendiculars are odd, use the last Method of the three.

The Map may be drawn, by knowing in what Parts of the Diagonal the Perpendiculars were taken, and thence setting them off, and joining their Extremity to those of the Diagonal.

The Area of the Square, Oblong, Rhombus, or Rhomboides, may be obtained in the like Manner, by a Diagonal and Perpendiculars.

If a Map of any of the foregoing Figures be given, with the Scale by which it was laid down, take the Dimensions from that Scale of Perches, and thereby adjust their Contents, without reducing them to Chains.

From this, and the sixth Problem, this following Method will be plain.

First Method to find the Area of Ground.

How the Area of a Piece of Ground, be it ever so irregular, may be obtained, by dividing it into Triangles, and Trapezia.

Plate VII. Fig. 4.

We here admit the Survey to be taken, and protracted; by having therefore the Map, and knowing the Scale by which it was laid down, the Content may be thus obtained.

Dispose the given Map into Triangles, by fine pencilled Lines, such as are here represented by pop'd Lines in the Scheme, and number the Triangles with 1, 2, 3, 4, *&c.* Your Map being thus prepared, rule a Table with four Columns; the first of which is for the Number of the Triangle, the second for the Base of it, the third for the Perpendicular, and the fourth for the Content in Perches.

Then proceed to measure the Base of Number 1. from the Scale of Perches the Map was laid down, and

and place that in the ſecond Column of the Table, under the Word Baſe; and from the Angle oppoſite to the Baſe, open your Compaſſes ſo, as when one Foot is in the Angular Point, the other being moved backwards and forwards, may juſt touch the Baſe Line, and neither go the leaſt above or beneath it; that Diſtance in the Compaſſes, meaſured from the ſame Scale, is the Length of that Perpendicular, which place in the third Column, under the Word Perpendicular.

If the Perpendiculars of two Triangles fall on one and the ſame Baſe, it is unneceſſary to put down the Baſe twice, but inſert the ſecond Perpendicular oppoſite to the Number of the Triangle in the Table, and join it with the other Perpendicular by a Brace, as No. 1 & 2, 4 & 5, 6 & 7, 9 & 10, *&c.*

Proceed after this Manner, 'till you have meaſured all the Triangles; and then by Prob. 6, find the Content in Perches of each reſpective Triangle, which ſeverally place in the Table oppoſite to the Number of the Triangle, in the fourth Column, under the Word Content.

But where two Perpendiculars are joined together in the Table, by a Brace, having both one and the ſame Baſe; find the Content of each, (being a Trapezium) in Perches, by Prob. 9, which place oppoſite the Middle of thoſe Perpendiculars, in the fourth Column, under the Word Content.

Having thus obtained the Content of each reſpective Triangle and Trapezium, which the Map contains, add them all together, and their Sum will

be the Content of the Map in Perches; which being divided by 160, gives the Content in Acres. Thus, for

EXAMPLE.

No.	Base	Perp.	Content
1	24.8	17.0	412.92
2		16.3	
3	28.2	16.0	225.6
4	39.8	19.6	712.42
5		16.2	
6	49.4	29.0	1086.8
7		15.0	
8	38.7	6.7	129.64
9	40.0	17.0	600.
10		13.0	
11	42.8	10.2	481.5
12		12.3	
13	26.2	17.9	234.49
14	24.0	11.6	259.2
15		10.0	
		Content in Perches	4142.57

This being divided by 160, will give 25A. 3R. 22P. the Content of the Map.

1) 17.0
2) 16.3

33.3
12.4

1332
666
333

412.92

6) 29.
7) 15.

44.

22.
49.4

988
988

1086.8

11) 10.2
12) 12.3

22.5
21.4

900
225
450

481.50

3) 28.2
8.

225.6

4) 19.6
5) 16.2

35.8
19.9

3222
3222
358

712.42

8) 38.7
6.7

2709
2322

259.29

129.64

13) 17.9
13.1

179
537
179

234.49

9) 17.
10) 13.

30.
20.

600

14) 11.6
15) 10.0

12.6
12.

259.2

Let

Let your Map be laid down by the largeſt Scale your Paper will admit, for then the Baſes and Perpendiculars can be meaſured with greater Accuracy than when laid down by a ſmaller Scale; and if poſſible meaſure from Scales divided diagonally.

If the Baſes and Perpendiculars were meaſured by Four Pole Chains, the Content of every Triangle, and Trapezium, may be had as before, in Problems 6 and 9, and conſequently the whole Content of the Map.

If any Part of your Map has ſhort or crooked Bounds, as thoſe repreſented in Plate VII, Fig. 5, then by the ſtreight Edge of a tranſparent Horn, draw a fine pencilled Line as AB, to ballance the Parts taken in and left out, as alſo another, BC: Theſe Parts when ſmall, may be ballanced very nearly by the Eye, or they may be more accurately ballanced by Method the Third. Join the Points A and C by a Line, ſo will the Content of the Triangle ABC, be equal to that contained between the Line AC, and the crooked Mearing from A to B, and to C: By this Method the Number of Triangles will be greatly leſſened, and the Content become more certain; for the fewer Operations you have, the leſs ſubject will you be to err; and if an Error be committed, the ſooner it may be diſcovered.

The Lines of the Map ſhould be drawn ſmall, and neat, as well as the Baſes; the Compaſſes neatly pointed, and Scale accurately divided; without all which, you may err greatly. The Multiplications ſhould be run over twice at leaſt, as alſo the Addition of the Column Content.

From

From what has been ſaid, it will be eaſy to ſurvey a Field, by reducing it into Triangles, and meaſuring the Baſes and Perpendiculars by the Chain. To aſcertain the Content only, it is not material to know at what Part of the Baſe the perpendicular was taken; ſince it has been ſhewn (in Cor. to Theo. 13. Sect. 1.) that Triangles on the ſame Baſe, and between the ſame Parallels are equal: But if you would draw a Map from the Baſes and Perpendiculars, it is evident that you muſt know at what Part of the Baſe the Perpendicular was taken in order to ſet it off in its due Poſition; and hence the Map is eaſily conſtructed.

Second Method to find the Area of Ground.

To determine the Area of a Piece of Ground, from the Map thereof, by the Square Inch Horn.

This Horn is tranſparent, and has on it a Square whoſe Side is an Inch; each Side of this is divided into 10 equal Parts, and the oppoſite Diviſions are joined by Right Lines, ſo the whole Square contains 10 times 10, or 100 ſmall Squares.

Let the Parallels, by which your Map is to be protracted, be an Inch aſunder, and at Right Angles to theſe, let other Parallels be drawn an Inch

asunder also; so will your Paper be divided into Square Inches. But great Care must be taken that the Parallels be exactly an Inch asunder, and that the cross Parallels be truly perpendicular to them, and also duly an Inch from each other.

When all your Parallels are drawn, lay the Edge of a straight Ruler diagonal wise, on the opposite Angles of a Square: Then if the Ruler's Edge cuts exactly the opposite Angles of every Square it passes over, and the Diagonals or Diameters of each be equal, the Paper is truly squared, otherwise not. And if the Parallels are an Inch asunder exactly, they will be all square Inches.

Having thus prepared your Paper. and on it either protracted or drawn your Map; lay your Inch Horn on every Square thro' which any of the Lines of your Map pass, and count the Number of small Squares on your Horn which lie within the Lines of the Map, which Number place down in that Part of a Square; do the like in every Part of a Square, and in every whole Square put down 100, or number them 1, 2, 3, *&c.* Add all those Numbers which you have in Parts of Squares and whole Squares together; and their Sum will be the Number of small Squares which the Map contains.

It is best to reckon the Parts of Squares without the Map as well as those within, and by so doing, if the Parts without and within be equal to 100, you may reasonably conclude that you have counted and estimated true.

Let ABCD, Plate VII. Fig. 7. be a Map, whose Area is required: Let the Paper be squared as before taught, and let every of the Squares in the

Scheme

Scheme represent square Inches. Let the Inch Horn, Plate VII. Fig. 6, be applied on every Square through which any of the Lines of the Map run, and count the Number of small Squares contained within the Lines of the Map, which put down on your Map, as in the Scheme: Then number the whole Squares separately, and add the small Squares contained in them, to the Number of small Squares in the Parts through which the Lines of the Map pass, and you have the Number of small Squares contained in the Map.

	$61\frac{1}{2}$	
25	2	
61	11	10
20	97	$78\frac{1}{4}$
21	99	35
$96\frac{1}{2}$	69	27
71	27	28
31	98	3
$98\frac{1}{4}$	28	47
22	28	52
31	11	4
$476\frac{3}{4}$	$531\frac{1}{2}$	$284\frac{1}{4}$

$476\frac{3}{4}$
$531\frac{1}{2}$
$284\frac{1}{4}$

$1192\frac{1}{2}$ small Squares in Parts of large ones.
1900 small Squares in 19 large ones

$3192\frac{1}{2}$ Content of the Map in small Squares.

Having obtained the Number of small Squares contained in your Map, you are next to consider

what Scale it was laid down by; and thence you may know what the Content of every ſmall Square is, and of courſe the Content of them all, or the Content of the Map. Thus;

If your Map be laid down by a Scale of 8 Perch to an Inch, the Side of the Square Inch will be 8, and its Content 8 Times 8, or 64 Perches; and becauſe there are 100 ſmall Squares in a large one, the Content of every ſmall Square will be 64 of a Perch; wherefore the Number of ſmall Squares being multiplied by .64 will give their Content in Perches, and thoſe divided by 160, give the Content in Acres.

If your Map be laid down by a Scale of 10 Perches to an Inch, the Content of every Square Inch will be ten Times 10, or 100 Perches; and ſince every Square Inch contains 100 ſmall Squares, therefore every ſmall Square will be 1 Perch; conſequently the Number of ſmall Squares contained in your Map, will be the Content of it in Perches.

If your Map be laid down by 15 Perch to an Inch, a Square Inch will contain 15 Times 15, or 225 Perches; and a ſmall Square being the hundredth Part of a large one, will contain 2.25 Perches; wherefore the Number of ſmall Squares being multiplied by 2.25 will give the Content in Perches; or to the double of the Number of ſmall Squares add $\frac{1}{4}$ of them, the Sum is the Content in Perches.

If your Map be laid down by 20 Perches to an Inch, the Square Inch will then contain 20 Times 20, or 100 Perches; and 1 ſmall Square will contain

contain

tain 4 Perches; wherefore the Number of ſmall Squares being multiplied by 4, gives the Content in Perches.

Or divide the Sum of the ſmall Squares, having cut off the laſt Figure, by 4; the Quotient is Acres, and what remains, if any, is a Rood or Roods: Then multiply your laſt Figure by 4, adding 1 for a Quarter, 2 for Half, or 3 for $\frac{3}{4}$ of a Square, (if you have any ſuch in the Sum of your ſmall Squares) and you will have the Perches. Thus the Number of ſmall Squares in the foregoing Map being $3192\frac{1}{2}$, I demand the Content in Acres.

$$319|2\tfrac{1}{2}$$

$$\text{Content } 79:3:10$$

Cut off the laſt Figure 2, and divide 319 by 4, the Quotient is 79 Acres, and the Remainder 3, is 3 Roods. Muliply the laſt Figure 2 by 4, and add in 2 for the half Square, and you have 10 Perches.

The Reaſon hereof is plain, when we conſider that any Number multiplied by 4, and divided by 160, will give in the Quotient the ſame, as if that Number was only divided by 40; or which is the ſame thing, cut off the laſt Figure for the 0, and divide by 4.

If your Map be laid down by 25 Perches to an Inch, every Square Inch will be 25 Times 25, or 625 Perches; wherefore every ſmall Square will be 6.25 Perches. The Number of ſmall Squares therefore

fore being multiplied by 6, adding $\frac{1}{4}$ of their Number to the Product, will give the Content in Perches.

If your Map be laid down by a Scale of 30 Perches to an Inch, every Square Inch will contain 900 Perches, and every ſmall Square 9 Perches; the Number of ſmall Squares therefore being multiplied by 9, will give the Content in Perches.

If your Map be laid down by a Scale of 35 Perches to an Inch, every Square Inch will contain 1225 Perches, and every ſmall Square 12.25 Perches; wherefore to 12 times the Number of ſmall Squares, add $\frac{1}{4}$ of their Number, and you have the Content in Perches.

If your Map be laid down by 40 Perches to an Inch, every Square Inch will contain 1600 Perches, and every ſmall Square 16 Perches; the Number of ſmall Squares therefore being multiplied by 16, gives the Content in Perches.

Or cut off the laſt Figure from the Sum of the ſmall Squares, and the other Figures are Acres; multiply that laſt Figure by 4, adding in 1, 2, or 3, for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, as before, and cut off 1 Figure to the Right from the Product, and the other, if any, are Roods: Multiply the laſt Figure by 4, and the Product is Perches.

For if any Number be divided by 10, which is done by cutting off the laſt Figure, it will quote the ſame as if that Number were multiplied by 16, and divided by 160.

Suppoſe

Suppoſe the Number of ſmall Squares in a Map be 3994 ½, required the Content in Acres.

Acres	39|94 ½
	4
Rood	1|8
	4
Perches	32

After the like Manner you may find the Content of any Map, by knowing the Scale by which it was laid down; for the Square of the Scale gives the Content of a Square Inch in Perches, the hundredth Part of which will be the Content of a ſmall Square.

I am confident I ſhall be judged to be too prolix on this Head; but as this Method has ſtood the Teſt of many Ages, and is ſtill uſed by many, I have for the Benefit of thoſe of the ſloweſt Capacity, explained it as fully as I was able.

Third

Third Method to determine the Area of Ground.

To determine the Area of a Piece of Ground, having the Map given, by reducing it to one Triangle equal thereto, and thence finding it's Content.

Plate VIII. Fig. 5.

Let A B C D E F G H be a Map of Ground, which you would reduce to one Triangle equal thereto.

Produce any Line of the Map, as AH, both Ways: Lay the Edge of a parallel Ruler from A to C, having B above it: Hold the other Side of the Ruler, or that next you fast; open 'till the same Edge touches B, and by it, with a Protracting Pin, mark the Point *b* on the produced Line: Lay the Edge of the Ruler from *b* to D, having C above it: Hold the other Side fast open, 'till the same Edge touches C, and by it mark the Point *c*, on the produced Line. A Line drawn from *c* to D will take in as much as it leaves out of the Map.

Again, lay the Edge of the Ruler from H to F, having G above it, keep the other Side fast, open 'till the same Edge touches G, and by it mark the Point *g*, on the produced Line: Lay the Edge of the Ruler from *g* to E, having F above it, keep the other Side fast, open, 'till the same Edge touches F, and by it mark the Point *f*, on the produced

duced Line. Lay the Edge of the Ruler from *f* to D, having E above it, keep the other Side faſt, open 'till the ſame Edge touches E, and by it mark the Point *e*, on the produced Line. A Line drawn from D to *e*, will take in as much as it leaves out. Thus have you the Triangle *c* D *e*, equal to the irregular Polygon A B C D E F G H.

If when the Ruler's Edge be applied to the Points A and C, the Point B falls under the Ruler, hold that Side next the ſaid Points faſt, and draw back the other to any convenient Diſtance; then hold this laſt Side faſt, and draw back the former Edge to B, and by it mark *b*, on the produced Line; and thus a Parallel may be drawn to any Point under the Ruler, as well as if it were above it. It is beſt to keep the Point of your Protracting Pin in the laſt Point in the extended Line, 'till you lay the Edge of the Ruler from it to the next Station, or you may miſtake one Point for another.

This may alſo be performed with a Scale, or Ruler, which has a thin ſloped Edge, called a Fiducial, or ſure Edge; and a fine pointed Pair of Compaſſes. Thus.

Lay that Edge on the Points A and C, take the Diſtance from the Point B to the Edge of the Scale, ſo as it may only touch it, in the ſame Manner as you take the Perpendicular of a Triangle; carry that Diſtance down by the Edge of the Scale parallel to it, to *b*; and there deſcribe an Arc on the Point *b*, and if it juſt touches the Ruler's Edge, the Point *b* is in the true Place of the extended Line. Lay then the Fiducial Edge of the Scale from *b* to D, and take a Diſtance from C, that will

just touch the Edge of the Scale; carry that Distance along the Edge, 'till the Point which was in C, cuts the produced Line in *c*; keep that Foot in *c*, and describe an Arc, and if it just touches the Ruler's Edge, the Point *c* is in the true Place of the extended Line. Draw a Line from *c* to D, and it will take in and leave out equally: In like Manner the other Side of the Figure may be ballanced by the Line *e* D.

Let the Point of your Compasses be kept to the last Point of the extended Line, 'till you lay your Scale from it to the next Station, to prevent Mistakes from the Number of Points.

That the Triangle *c* D *e*, is equal to the right-lined Figure A B C D E F G H, will be evident from Problems 18, 19. Sect. 1: For thereby, if a Line were drawn from *b* to C, it will give and take equally, and then the Figure *b* C D E F G H, will be equal to the Map. Thus the Figure is lessened by one Side, and by the next Ballance Line will lessen it by two, and so on, and will give and take equally. In the same Manner an Equality will arise on the other Side.

The Area of the Triangle is easily obtained, as before, and thus you have the Area of the Map.

It is best to extend one of the shortest Lines of the Polygon, because if a very long Line be produced, the Triangle will have one Angle very obtuse, and consequently the other two very acute; in which Case it will not be easy to determine exactly the Length of the longest Side, or the Points where the Ballancing Lines cut the extended one.

This

This Method will be found very uſeful and ready in ſmall Encloſures, as well as very exact; it may be alſo uſed in large ones, but great Care muſt be taken of the Points on the extended Line, which will be crowded, as well as of not miſſing a Station.

Fourth Method to determine the Area of Ground.

To determine the Area of a Piece of Ground, having the Map thereof given by a Scale of Acres.

In this Caſe let your Parallels be half an Inch aſunder, and (if it will admit it) let your Map be laid down by a Scale of 20 Perches to an Inch: Provide yourſelf with a tranſparent Horn, with Parallels on it at half an Inch aſunder, to which the Ends of the Horn muſt be at right Angles; an Inch Horn may be made to ſerve this Purpoſe. Between two of the middle Parallels let a fine parallel Line be drawn, equi-diſtant to each, or a Quarter of an Inch from either. Apply the middle Parallels on the Horn between any two Parallels on the Map, and bring the End of the Horn ſo to the Lines of the Map between the ſaid Parallels, that you may have as much under it, as you eſtimate to be without it; and with a fine Pencil Point, or Protracting Pin, draw a Line between the two Parallels by the End of the Horn. You muſt be careful in keeping the Parallels on the Horn exactly over the Parallels on your Map. Do the like by the Lines of the

the Map between all the Parallels on it, till you have reduced what was between them into equivalent right-angled Parallelograms.

Plate VIII. Fig. 1.

Let ABCD represent a Map of Ground which is laid down by a Scale of 20 Perches to an Inch, and let the parallel Lines represented in the Scheme, be ſuppoſed to be half an Inch aſunder: By this Hypotheſis therefore it will be plain, that the Breadth between any two Parallels will be 10 Perches; and becauſe 160 Perches make an Acre, it follows, that 16 Perches in Length between any two Parallels will be an Acre.

Plate VIII. Fig 3.

Make a Scale of equal Parts, whoſe Diviſions ſhall be 16 Perches, or 8 Tenths of an Inch aſunder; and ſubdivide one of thoſe Diviſions into Tenths, and half Tenths; The Parts between may be eſtimated near enough. Or one of the Diviſions may be divided diagonally into 100 equal Parts. Number the firſt large Diviſion 1, the ſecond 2, the third 3, *&c.* ſo every one of the large Diviſions will be an Acre, and the ſmall ones will be decimal Parts of an Acre.

Plate VIII. Fig. 2. repreſents the two middle Parallels on the Horn, with the intermediate Parallel at a Quarter of an Inch from each.

Apply this Part of the Horn between any two Parallels on the Map, Fig. I. 'till by the Edge thereof you find by Eſtimation you have as much at the Boundaries of the Map under it, as you leave

leave out, and by the End of the Horn draw a Ballance Line with a Pencil, as before, and do the like between every two Parallels. The Pricked Lines in the Scheme represent the Ballance Lines.

Where the Boundaries are very crooked, as between *a* and *b*, ballance them with the Line *ab*, by the foregoing Method; bring the Edge of the middle Parallels on the Horn to this Line, 'till the intermediate Line meets *ab*, in *c*; then by the said Edge draw the Line *de*, and the Part left out will be equal to that taken in; which must be manifest by what has already been said. And this Method may be used wherever the Parts left out, and taken in, are too great to be estimated.

If an oblique Right Line be the Boundary between two Parallels, as the Line B*f* is, bring the Edge of the Middle Parallels on the Horn to this Line, 'till the intermediate Line meets it; and by the Edge draw the Ballance Line.

Having reduced what lies between every two Parallels to an Oblong equal thereto, number them with 1, 2, 3, 4, *&c.*

Then either apply the Fiducial Edge of your Scale where the Divisions of Acres are, to find the Length of each Oblong; or measure the Lengths from your Scale of Acres, which you have divided diagonally: Set these severally opposite to their respective Numbers, and their Sum will be the Acres and decimal Parts of an Acre contained in all the Oblongs. Thus,

No.			
1	.82		
2	2.23		
3	3.15		
4	4.08		
5	3.95		
6	1.92		
7	.92		
	17.07		
	4		
	.28		
	.40		A. R. P.
	11.20	Content	17. 0. 11.

If there be a ſmall Piece above or below, that won't reach a Parallel, or if there be any long irregular Excreſcences on either Side, let theſe be caſt up by Triangles, and their Content be added to the Sum of the Oblongs, for the Content of the Map.

If your Map be laid down by a Scale of forty Perches to an Inch, it is beſt to have Quarter Inch Parallels, and then every Acre will be 4 Tenths of an Inch in Length, which may be laid on the other Edge of the Scale.

Let your Map be laid down by what Scale ſoever, if you caſt it up by the large Scale of Acres, or by that of 20 Perches to an Inch, the true Content may be eaſily obtained, as will hereafter be ſhewn.

The

The larger your Scale is by which your Map is laid down, the more certain the Content will be by any Method whatſoever.

When a Perſon is ſome Time accuſtomed to this Method, he will prefer it to any of the foregoing for its Expedition and Certainty.

If you be unprovided with a Scale of Acres, you may meaſure the Lengths of the Oblongs by the Diviſions on the Edge of the Protractor; and if the Map be ruled with half Inch Parallels, the Sum of the Lengths of the Oblongs being divided by 16, will give the Content in Acres, if the Map was laid down by 20 Perches to an Inch; but if by 40, divide the Sum by 4, and you have the Content required.

Fifth Method to determine the Area of Ground.

To determine the Area of a Piece of Ground, having the Map given, by Weight.

Let the Map, Plate VIII. Fig. 6. be that whoſe Area is required.

Let Parallels be drawn at half an Inch aſunder, and others at Right-Angles to thoſe at a like Diſtance; ſo will each Square be a Quarter of a Square Inch in Area.

With

With a Pair of fine Sciſſars, or a Penknife, cut away all the Squares marked *a*, which the Bounds of the Map do not reach, as unneceſſary and uſeleſs; then will remain all the whole Squares contained within the Body of the Map, and the Squares which the Bounds paſs through, which are mark'd z, z, z, *&c.*

Count the Number of whole Squares in the Body of the Map, and thoſe marked z, together, which Number note down; then find with Grains and Tenths of a Grain, the Weight of all that Paper. Call the Number of Squares, firſt Area, and their Weight in Grains and Parts, call their firſt Weight.

Cut the Map then cloſe by the Bounds, ſo will the Pieces of Squares without the Bounds, marked z, be cut away, and the Map only remain, which weigh in Grains and Decimals of a Grain, as before. Then ſay, as the firſt Weight is to the firſt Area, ſo is the ſecond Weight to the ſecond Area, which gives the Area of the Map in Squares, and Decimals of a Square. Then by knowing by what Scale it was laid down, the Area of each Square is known, and conſequently the Area of the whole Map. The Area of each Square by the different Scales moſt uſed, will be thus:

By a Scale of	Perch to an Inch, every Square will be	
8	16	Perches.
10	25	Perches.
16	64	Perches.
20	100	Perches.
40	400	Perches.
80	10	Acres.

The following was an Experiment made a few Years since upon the Map, Plate X. Fig. 3, which when caſt up by the following univerſal Methods, gives 110A. 2R. 32P. The Map was laid down by a Scale of 20 Perches to an Inch.

Number of Squares in the Map, and thoſe marked z, 223.

	Grains.
Weight of the Paper	74.6
Weight of the Map	59.2

Then 74.6 : 223 : : 59.2 : 176.96 Squares in the Map, and each Square being 100 Perches, makes 17696 Perches, or 110A. 2R. 16P. differing only 16 Perches from the true Content.

This may alſo be performed by drawing a Trapezium about the Map, and finding its Content as before, cutting it out and weighing it, and afterwards the Map; by ſaying, as the Weight of the Trapezium is to the Content of the Trapezium, ſo is the Weight of the Map to the Content of the Map. But this Method will not be ſo exact as the foregoing one; for there will be a much greater Space, or Area comprehended between the Trapezium and the Map, than that contained in the Parts of Squares thro' which the Lines of the Map run, and the Map; therefore, as there are ſome Inequalities in all Paper, the greater the Error will be.

Tho' this Method of Weighing may ſeem whimſical and ridiculous, yet if Experiments be made with nice Scales and Weights, upon Maps laid down by large Scales, and drawn upon good even *Writing-Demy* Paper, the Contents produced will be found to be much nearer the Truth than can be imagined.

SECT.

SECT. V.

Containing Four new concise Methods of determining the Areas of Right Lined Figures universally, or by Calculation: With some necessary previous Definitions and Theorems.

CALCULATION.

DEFINITIONS.

Plate VIII. Fig: 7.

I. MERIDIANS are North and South Lines, which are supposed to pass through every Station, running parallel to each other.

II. The Difference of Latitude, or the Northing or Southing of any stationary Line, is the Distance that one End of the Line is North or South from the other End; or it is the Distance which is intercepted on the Meridian, between the Beginning of the stationary Line, and a Perpendicular drawn from the other End to that Meridian. Thus, if NS be a Meridian Line passing through the Point A of the Line AB, then is A*b* the Difference of Latitude, or Southing of that Line.

 III. The

III. The Departure of any ſtationary Line, is the neareſt Diſtance from one End of the Line, to a Meridian paſſing through the other End. Thus, B*b* is the Departure, or Eaſting of the Line AB.

But if CB be a Meridian, and the Meaſure of the ſtationary Diſtance be taken from B to A; then is BC the Difference of Latitude, or Northing, and AC the Departure, or Weſting of the Line BA.

IV. That Meridian which paſſes through the firſt Station, is ſometimes called the firſt Meridian; and ſometimes it is a Meridian paſſing on the Eaſt or Weſt Side of the Map, at the Diſtance of the Breadth thereof, from Eaſt to Weſt, ſet off from the firſt Station.

V. The Meridian Diſtance of any Station is the Diſtance thereof from the firſt Meridian, whether it be ſuppoſed to paſs through the firſt Station, or on the Eaſt or Weſt Side of the Map.

THEO. I.

In every Survey which is truly taken, the Sum of the Northings will be equal to that of the Southings; and the Sum of the Eaſtings equal to that of the Weſtings.

Plate IX. Fig. 1.

Let *a b c e f g* repreſent a Plot, or Parcel of Land. Let *a* be the firſt Station, *b* the ſecond, *c* the third, &c. Let NS be a Meridian Line, then will all Lines

Lines parallel thereto, which paſs through the ſeveral Stations, be Meridians alſo; as *ao*, *bs*, *cd*, &c. and the Lines *bo*, *cs*, *de*, &c. perpendicular to thoſe, will be Eaſt or Weſt Lines, or Departures.

The Northings $ei + go + bq = ao + bs + cd + fr$ the Southings. Thus:

Let the Figure be compleated; then it is plain that $go + bq + rk = ao + bs + cd$, and $ei - rk = fr$. If to the former Part of this firſt Equation $ei - rk$ be added, and *fr* to the latter, then $go + bq + ei = ao + bs + cd + fr$; that is, the Sum of the Northings is equal to that of the Southings.

The Eaſtings $cs + qa = ob + de + if + rg + oh$, the Weſtings. Thus:

For $aq + yo$ (az) $= de + if + rg + ob$, and, $bo = cs - yo$. If to the former Part of this firſt Equation $cs - yo$ be added, and *bo* to the latter, then $cs + aq = ob + de + if + rg + ob$; that is, the Sum of the Eaſtings is equal to that of the Weſtings. Q. E. D.

SCHOLIUM.

This Theorem is of Uſe to prove, whether the Field-Work be truly taken, or not: For if the Sum of the Northings be equal to that of the Southings, and the Sum of the Eaſtings be equal to that of the Weſtings, the Field-Work is truly taken, otherwiſe not.

Since the Proof and Certainty of a Field-Work depend on this Truth, it will be neceſſary to ſhew how the Difference of Latitude, and Departure, for

for any ſtationary Line, whoſe Bearing and Diſtance are given, may be obtained by Trigonometry, or by Tables hereunto annexed.

PROBLEM.

To find the Difference of Latitude and Departure, the Bearing and ſtationary Diſtance being given. by Trigonometry.

Plate IX. Fig. 1.

Let the Bearing of the Diſtance Line *ab*, be SW $16^{\circ}\frac{3}{4}$, and its Length in Four-Pole Chains, 10C. 12L. Required *ao*, the Difference of Latitude, and *ob*, the Departure.

By Caſe 1. of *Rectangular Trigonometry*.

R :	*ab* : :	S. *oab* :	*bo*	
90°	10.12	$16^{\circ}.45^{1}$	2.92	Dep.
R. :	*ab* : :	S. *abo* :	*ao*	
90°	10.12.	$73^{\circ}.15^{1}$	9.69	Diff. Lat.

Thus the Difference of Latitude, or Southing is found to be 9C. 69L. and the Departure, or Weſting, 2C. 92L. And in like Manner the Difference of Latitude and Departure may be found for any ſtationary Diſtance, having the Bearing, and Length thereof given.

This may alſo be performed by *Gunter*'s *Scale*, as already ſhewn, in Caſe 1. of *Rectangular Trigonometry*.

2. *To find the Difference of Latitude and Departure, by Help of the annexed Table.*

In the first Column find the Degree, and the $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of a Degree contained in the Bearing; in a Line with which, under N. S and E. W. you have the Difference of Latitude, and half the Departure for that Bearing, the Distance being 1 Four-Pole Chain.

Therefore, if the corresponding Numbers, opposite to the Degrees in the Table, be severally multiplied by the Length of the stationary Distance, their respective Products will be the Difference of Latitude, and Half Departure for that Line. Thus,

Let the Bearing as before be SW. 16°$\frac{3}{4}$, Distance 10 C. 12 L. Required the Difference of Latitude and Departure.

Opposite to 16°$\frac{3}{4}$, are these Numbers.

.9576	N. S.	.1441	E. W.
10.12		10.12	
19152		2882	
9576		1441	
9576		1441	
9.690912	Southing	1.458292	Half Westing.
		2.916584	Westing.

THEO

THEO. II.

If a Map lies to the East of any first Meridian, the farther any Point be taken in the several stationary Distances that have East Departure, the farther will that Point be from the first Meridian; but the farther any Point be taken in stationary Distances that have West Departure, the nearer that Point will be to the first Meridian.

Plate IX. Fig. 2.

Let the first Meridian be N. S, and let the Stations *a*, *b*, *c*, lie on the East Side thereof; let the Perpendiculars *af*, *bm*, *cn*, and *dk* be drawn; draw *ao*, *bi*, and *cp* parallel to NS.

The Departure of the first Line *ab* is *ob* and lies to the West Side of its Meridian *ao*, which passes through the Station *a*: But the Point *b*, or any Point in the Line *ab*, is nearer to the first Meridian NS, than the Point *a* is; for *af* (*om*) ---*ob*=*bm*, therefore *bm* is less than *af*, and consequently the Point *b* is nearer N. S, than the Point *a* is.

The Departure of the next Station is *ci*, which is to the Eastward of its Meridian *bi*, which passes through the Station *b*; but the Point *e*, or any Point in the Line *bc*, is more remote from the first Meridian NS, than the Point *b* is; for *bm*+*ci*=*cn*.

Therefore

Therefore when the Departure of any ſtationary Diſtance is Eaſt, the further you proceed on in the Diſtance Line, the more you remove from the firſt Meridian. But when the Departure is Weſt, the farther you proceed on in the Diſtance Line, the nearer you approach the firſt Meridian. Q. E. D.

COR. I.

Hence if the Diſtance of the firſt Station from the firſt Meridian be known, and alſo the ſeveral Departures *ob*, *ci*, *pd*; the Diſtances of the Points *b*, *c*, and *d*, from that Meridian may be likewiſe known: For the firſt Departure being Weſt, $af - bo = bm$; the next Departure *ic* being Eaſt, $bm + ci = cn$; and the next Departure being Weſt, $cn - pd = dk$. And if you proceed in the like Manner to add the Eaſt, and ſubtract the Weſt Departure in any Survey, the Diſtance of any Station from the firſt Meridian may be known.

COR. II.

As by the whole Departure of each ſtationary Diſtance, with the Diſtance of any one Station from the firſt Meridian, the Diſtance of every Station in the Map from the firſt Meridian may be known; ſo by having Half the Departure of each ſtationary Line given, with the Diſtance of the firſt (or any other) Station from the firſt Meridian, the Diſtance of the Middle of each ſtationary Line, from the firſt Meridian may be found thus. If *bq* be Half the Departure of the firſt Station, the Point *b* will be in the Middle between a and *b*; for *ob* : *ba* : : *bq* : *ba*, but *ob* being double of *bq*, *ba* will be double of *ba*.

Again, *ob* : *oa* : : *bq* : *qa*, wherefore *oa* will be double of *qa*, and *qg* will fall in the Middle between *af* and *bm*: And for the ſame Reaſon admitting *lz* to be Half the Departure of the ſecond ſtationary Line; *le* will fall in the Middle between *bm* and *cn*; and conſequently if Half the Departure of each Station be given, the Lines *gb*, *mb*, *el*, *nc*, &c. will alſo be given, for *af*---*qb*=*bg*. *bg*---*bq* (*kb*)=*bm*. *bm*+*lz* =*le*. *le*+*lz* (*xc*)=*cn*. *cn*---*ry*=*rs*. *rs*---*ry*=*dk*.

LEMMA.

In every Trapezium ABCD, having two Sides, AD, BC, perpendicular to the ſame Side AB, Half the Sum of thoſe perpendicular Sides, viz. IF, multiplied into AB, the Baſe on which they ſtand, will be the Area of the Trapezium ABCD.

Plate VIII. Fig. 10.

Let DK be drawn parallel to AB, and let it be biſected by the Perpendicular FHI; it is plain, from ſimilar Triangles, that it will alſo biſect DC in F: Through F, draw FG parallel to DK, and this will alſo biſect CK in G.

It is manifeſt that KC is equal to the Difference of the Sides BC, AD, and GK = GC = ED their Half Difference: Therefore (by Cor. to Theo. 4. Sect. 2.) CB--CG=GB=FI, will be half the Sum of AD and CB; and it has been ſhewn that DFE, GFC are equilateral, and hence they are equal to each other. But AB multiplied by FI, or BG, Half the Sum of the Sides AD, and BC, will be the

the Area of the Rectangle AEGB=ABCD: For if from the Trapezium ABCD, the Triangle FGC be taken, and in lieu thereof DEF be added, the Rectangle AEGB, which is made of Half the Sum of the Sides into the Base, will be equal to the Trapezium ABCD. Q. E D.

THEO. III.

If the Meridian Distance taken in the Middle of every stationary Line, be multiplied into the particular Northing or Southing of that Line; and if the Difference between the Sum of the North and the Sum of the South Products be taken, it will be the Area of the Survey.

Plate IX. Fig. 3.

Let ABCDEFGA be the Map of a Survey whose Area is required.

To the first Meridian NS, drawn on the West Side of the Map, let Perpendiculars be drawn from the Beginning and Middle of every stationary Line; let also Meridians be drawn through each Station, and the Figure be compleated.

The Northings are A*b*=WM, B*d*=MN, F*q*=QS, and G*g*=SW.

The Meridian Distances to the Middle of every stationary Line tending to the North, are IY, KO, P*a* and HV.

The Northings (F*q*) QS multiplied by P*a*=Area of FGSQF, (by the Lemma); and (G*g*) SW multiplied by HV=Area SGAWS. (A*b*) WM multiplied by IY= Area WABMW, and (B*d*) MN multiplied by KO=Area MBCNM; the Sum of all these Areas in the whole North Area QFGABCNQ.

In the same Manner the Southings (C*b*) NZ multiplied by LL=Area NCDZN, (D*m*) ZT multiplied by MX = the Area ZDETZ, (E*n*) TQ multiplied by RR=the Area TEFQT, the Sum of all these Areas is the South Area NCDEFQN.

The Area of NCDEFQN=QFGABCNQ=Area of the Map ABCDEFA. Q. E. D.

The Demonstration of this Theorem in a more complicated Figure.

Plate IX. Fig. 4.

Let *a* be the first Station, *b* the second, *c* the third, *&c.* and the Line NS the first Meridian.

The Figure made of the several Southings, into the Meridian Distances to the Middle of every stationary Line is *ocdefghot kestk*. And the Figure made of the several Northings into the Meridian Distance to the Middle of every stationary Line, is *hgabcobt kestk*.

The Difference of these Areas is *ocdefgho*---*hgabcob*=*abcdefga*, the Area of the Map. Q. E. D.

If

If the firſt Meridian *ob* had paſſed either through the extreme Weſt Point *a*, or the Eaſternmoſt Point *g*; the Demonſtration would be the ſame, both in this and the laſt Caſe.

SCHOLIUM.

From theſe Theorems we learn how to find the Area of a Piece of Ground by Calculation or without a Map, when the firſt Meridian paſſes on the Eaſt or Weſt Side of a Map.

To find the Area of Ground from the Field-Notes, by Calculation, or without the Aſſiſtance of a Map.

In the following Field Book, Method No. 1.

Transfer the Letters of the Bearing which are in the ſecond Column, from thence to the fourth, and place the firſt Letter thereof over the ſecond: Thus, the firſt Bearing being NE, in the fourth Column, or that of Latitude and Departure, write down the Letter N, and under it E; the firſt ſhews that the Difference of Latitude is Northing, and the ſecond that the Departure is Eaſting: The like is done with the ſecond Bearing, being NE alſo. The third Bearing being Eaſt, has neither Northing or Southing; therefore in the Place of either ſet down Cyphers only. The fourth Bearing being SW, write down S over W; and ſo proceed to ſet down the initial Letters of every Bearing one over the other: But if a Bearing be N, S, E, or W, put

put down such a Letter, and Cyphers in the Place of the second Letter.

Before or after every Letter E, place the Sign +; and also before or after every W, place the Sign—; to shew that every Easting is to be added, and every Westing substracted.

By Trigonometrical Calculation, or from the annexed Tables, find the Difference of Latitude an Half Departure of each Line, and place them after their respective Letters. Thus the first Bearing being N. E. 75, Distance 13 C. 70 L. the Northing will be 3 C. 54 L. and half the Easting 6 C. 6 L. the former of which place after the Letter N and the latter after E: and so proceed with all the rest, 'till the Column of Difference of Latitude and Departure be compleat.

N. B. It has been already shewn, that the Numbers under the Column EW, in the annexed Table, are half the Eastings or Westings, or half the Departure; but for Brevity sake, we shall hereafter call them Eastings or Westings, and sometimes Departures.

On a loose Piece of Paper write down the Letters N, S, E, and W, at any convenient Distance asunder; and from the Column of Latitude and Departure collect all the Northings, Southings, Eastings, and Westings; add them up severally, and if the Northings are equal to the Southings, and the Eastings to the Westings, the Work is truly taken, otherwise not, as before demonstrated.

The

The Latitudes and Departures in the following Field Book, are thus collected.

N.	S.	E.	W.
3.54	29.44	6.61	9.74
9.05	3.87	1.80	7.76
9.00	1.21	8.10	5.77
6.94	2.75	5.74	4.64
15.38	10.48	2.68	1.40
12.93	9.69	5.84	1.46
57.44	57.44	30.77	30.77

To find the Numbers for the Column entitled Meridian Distance.

Opposite Number 0 in your Table, place 61 C. 54 L. the whole Departure, or double the Sum of the half Eastings, or half Westings; in the Column of Meridian Distance.

In Plate IX. Fig. 3. Let AW represent this first Number, *viz.* 61 C. 54 L. and NQ the first Meridian Line; and since the Map lies to the East Side of that Meridian, (by Theo. 3. of this Section) those Lines that have East Departure will lie farther from the first Meridian, than those that have West Departure; and therefore knowing the Length of the Line AW, the Lengths of the other Lines IY, BM, *&c.* may be found, by adding the Eastings, and substracting the Westings, by Cor. 2. Theo. 2. of this Section.

The first Meridian is supposed to be the Length of the whole Departure, or the entire Easting,

or Weſting, from the firſt Station: For ſhould the firſt Station be at the Eaſtermoſt Point of the Land, the firſt Meridian will then paſs thro' the Weſtermoſt, and the Map will be entirely on the Eaſt of the firſt Meridian: But were the Meridian Diſtance leſs than whole Eaſting or Weſting, and the firſt Station at the Eaſtermoſt Point of the Land, then it is plain, that the firſt Meridian will paſs through the Map, and Part thereof will lie on the Eaſt, and the reſt on the Weſt of the ſaid firſt Meridian. In this Caſe therefore no Number leſs than the whole Eaſting or Weſting will be ſufficient for the Meridian Diſtance, in order that the Map ſhould lie entirely on the Eaſt thereof. But if the firſt Station be not the Eaſtermoſt Point of the Land, a leſs Number might be ſufficient for the Meridian Diſtance, but how much leſs would be troubleſome and needleſs to determine; ſince the Meridian Diſtance being made equal to the entire Eaſting or Weſting will anſwer in all Caſes, let the firſt Station be where it will; as it leaves the Map entirely to the Eaſt of the firſt Meridian.

The ſame Reaſoning will hold good, if you would have the Map to lie entirely on the Weſt of the firſt Meridian.

In the following Table or Field-Book the Eaſting of the firſt ſtationary Line is marked + 6.61, muſt be added to 6.54, and placed in a Line with N 3.54, and 68.15 + 6.61 = 74.76 muſt be placed in a Line with E+6.61.

Again, 74.76+1.80=76.56, which place in a Line with 9.65; and 76.56+1.80=78.36, which place in a Line with E+1.80. Proceed in the like Manner to add the Eaſtings, and ſubſtract the Weſt-

ings,

ings, 'till you have compleated the Column of Meridian Diſtance; the laſt Number of which will be equal to the firſt, if the Work be right, becauſe there was juſt as much added, as ſubſtracted, ſince the Sum of the Eaſtings is equal to that of the Weſtings.

The Numbers in the Column of Meridian Diſtance which are in the ſame Line with the Northings or Southings, in the Column of Latitude and Half Departure, are the Meridian Diſtances in the Middle of each Line; and thoſe in the ſame Column which are in the Line with the Eaſtings or Weſtings, are the Meridian Diſtances at every Station, or at the Beginning of every Line: And ſince it has been demonſtrated in Theo. 3. of this Sect. that if the Meridian Diſtances taken in the Middle of every Line, be multiplied into the particular Northing or Southing of that Line, that the Difference of the Sums of the North and South Products, will be the Area of the Map; it follows, that the upper Number of every two which ſtand in the Column of Meridian Diſtance, muſt be multiplied by the Northing or Southing adjoining it. Theſe ſeveral Products muſt be put in the Columns of N. Area, or S. Area, according as the Difference of Latitude is North or South. This if 68.15, the Meridian Diſtance to the Middle of the firſt Line, be multiplied by 3.54, the Northing adjoining it, the Product 241.2510, muſt be put in the Column of North Area; but in the 4th Sationary Line, when 84.82 is multiplied by 29.44, the Southing of that Line, the Product 2497.1008 muſt be placed in the Column of S. Area, and ſo of the reſt.

Laſtly, if you add up the Columns of N. and S. Area; the Difference of theſe will be the Area of the Survey in ſquare Four-Pole Chains, which may be eaſily reduced into Acres as before.

Thus the Content of the following Field-Book is 1107.0513 ſquare Four-Pole Chains, or 110.70513 Acres.

Acres	110.70513
	4
Roods	2.82052
	4
Perches	32.8208

A. R. P.
Or 100. 2. 32. or nearly 33 Perches.

FIELD

No. Sta.	Bearings	C. L.	Latit. and Half Dep.	Mer. Dist.	N Area.	S Area.
0				61.54		
1	NE 75	13.70	N 3.54 E+ 6.61	68.15 74.76	241.2510	
2	NE 20½	10.30	E 9.65 E+ 1.80	76.56 78.36	738.8040	
3	East	16.20	0.00 E+ 8.10	86 46 94.56		
4	SW 33½	35.30	S 29 44 W--- 9 74	84.82 75.08		2497.1008
5	SW 76	16.00	S 3.87 W--- 7.76	67.32 59.56		260.5284
6	North	9.00	N 9.00 0.00	59.56 59.56	536.0400	
7	SW 84	11.60	S. 1.21 W--- 5.77	53.79 48.02		65 0859
8	NW 53¼	11.60	N 6.94 W-- 4.64	43.38 38.74	301.0572	
9	NE 36¾	19.20	N 15 38 E+ 5.74	44.48 50.22	684.1024	
10	NE 22½	14.00	N 12.93 E+ 2.68	52 90 55 58	683.9970	
11	SE 76¾	12 00	S 2.75 E+ 5.84	61.42 67.26		168.9050
12	SW 15	10.85	S 10 48 W--- 1.40	65.86 64.46		690 2128
13	SW 16¾	10.12	S. 9 69 W--- 1.46	63.00 61 54		610.4700
					3185.2516	4292.3029
					Sub	3185.2516
					Area in square Four-Pole Chains	1107 0513
					Area in Acres and Decimal Part	110.70513

The foregoing FIELD-BOOK, *by a second Method.*

The Content of the foregoing Field-Book may as well be found by putting† to the Westings, and --- to the Eastings; that is, by adding the Westings and subtracting the Eastings, which is plain, if you conceive the first Meridian to be on the East Side of the Map. Thus,

No. Sta.	Bearings.	C. L.	Latit. and Half Dep.	Mer. Dist.	N. Area.	S. Area.
0				61.54		
1	NE 75	13.70	N 3.54 E—6.61	54.93 48.32	194.4522	
2	NE 20½	10.30	N 9.65 E—1.80	46.52 44.72	448.9180	
3	East.	16.20	0.00 E—8.10	36.62 28.52		
4	SW 33½	35.30	S 29.44 W+9.74	38.26 48.00		1126.3744
5	SW 76	16.00	S 3.87 W+7.76	55.76 63.52		215.7912
6	North.	9.00	N 9.00 0.00	63.52 63.52	571.6800	
7	SW 84	11.60	S 1.21 W+5.77	69.29 75.06		83.8409
8	NW 53¼	11.60	N 6.94 W+4.64	79.70 84.34	553.1180	
9	NE 36¾	19.20	N 15.38 E— 5.74	78.60 72.86	1208.8680	
10	NE 22½	14.00	N 12.93 E— 2.68	70.18 67.50	907.4274	
11	SE 76¾	12.00	S 2.75 E—5.84	61.66 55.82		169.5650
12	SW 15	10.85	S 10.48 W+ 1.40	57.22 58.62		599.6656
13	SW 16¾	10.12	S 9.69 W+1.46	60.08 61.54		582.1752
					3884.4636	2777.4123
				Subtract	2777.4123	
Content as before in			Four-Pole Chains		1107.0513	
			Content in Acres		110.70513	

Remarks on the two foregoing Methods.

1. That the Sum of the South Area Column in the firſt, and North Area in the ſecond, are the greateſt.

2. The Sum of the North Area Column in the firſt, added to the Sum of the North Area Column in the ſecond, is equal to the Sum of the South Area Column in the firſt, added to the Sum of the South Area Column in the ſecond. Thus,

1.	N. Area Column	3185.2516
2.	N. Area Column	3884.4636
	Sum	7069.7152

1.	S. Area Column	4292.3029
2.	S. Area Column	2777.4123
	Sum	7069.7152

3. That if the Sum of the North Area Column of the firſt, be added to the Sum of the South Area of the ſecond, and this Sum be taken from a Product made by the Multiplication of twice the whole Meridian Diſtance, or twice the entire Eaſting or Weſting, into the Sum of the whole Northing or Southing, the Remainder will be the Area of the Survey. Thus,

Firſt Meridian Diſtance, or the entire Eaſting or Weſting, is 61.54, the Double of which is

123.

123.08. The Sum of the whole Northing or Southing is 57.44. Then,

123.08 multiplied by 57.44	=7069.7152
3185.2516 + 2777.4123	=5962.6639
Area of the Survey as before	1107.0513

4. If the Sum in the South Area Column in the firſt, be added to the Sum of the North Area Column in the ſecond, and from that Sum be ſubſtracted the former Product 7069.7152, the Remainder will be the Area of the Survey.

Sum of the South Area of the firſt	4292.3029
Sum of the North Area of the ſecond	3884.4636
The Sum	8176.7665

The Sum	8176.7665
The Product before found	7069.7152
Area as before	1107.0513

THEO.

THEO. IV.

When the first Meridian passes thro' the Map.

If the East Meridian Distances in the Middle of each Line be multiplied into the particular Southing, and the West Meridian Distances into the particular Northing, the Sum of these Products will be the Area of the Map.

Plate X. Fig. 1.

Let the Figure *abkm* be a Map, the Lines *ab*, *bk* to the Southward, and *km*, *ma* to the Northward, NAS the first Meridian Line passing through the first Station *a*.

The Meridian Distances East $\left\}\begin{matrix} zd \times ao \\ tu \times ox(by) \end{matrix}\right\} = \text{Area} \left\{\begin{matrix} am \\ aw \end{matrix}\right.$

The Meridian Distances West $\left\}\begin{matrix} ef \times gx \\ hh \times ga(my) \end{matrix}\right\} = \text{Area} \left\}\begin{matrix} xp \\ gl \end{matrix}\right.$

These four Areas $am + ow + xp + gl$ will be the Area of the whole Figure *cmswiprlc*, which is equal to the Area of the Map *abkm*. Compleat the Figure.

The Parallelograms *am* and *ow*, are made of the East Meridian Distances *dz* and *tu*, multiplied into the Southings *ao* and *ox*. The Parallelograms *xp* and *gl* are composed of the West Meridian Distances

tances *ef* and *hh*, multiplied into the Northings *xg* and *ga*(*my*) but these four Parallelograms are equal to the Area of the Map; for if from them be taken the four Triangles marked Z, and in the Place of those be substituted the four Triangles marked O, which are equal to the former; then it is plain the Area of the Map will be equal to the four Parallelograms. Q. E. D.

THEO. V.

If the Meridian Distance when East, be multiplied into the Southings, and the Meridian Distance when West be multiplied into the Northings, the Sum of these less by the Meridian Distance when West, multiplied into the Southings, is the Area of the Survey.

Plate X. Fig. 2.

Let *a b c* be the Map.

The Figure being compleated, the Rectangle *af* is made of the Meridian Distance *eq*, when East multiplied into the Southing *an*; the Rectangle *yk* is made of the Meridian Distance *xw*, multiplied into the Northing *cz* or *ya*. These two Rectangles, or Parallelograms, *af*+*yk*, make the Area of the Figure *dfnyikd*, from which taking the Rectangle *oz*, made of the Meridian Distance *tu*, when West into the Southing *ob* or *bm*, the Remainder is the Area of the Figure *dfobikd*, which is equal to the Area of the Map.

Let *bou*$=$Y, *urib*$=$L, *ric*$=$C, *wrc*$=$Z, *akw* $=$K and *efb*$=$B, *ade*$=$A. I ſay, that $Y+Z+B = K+L+A$.

$Y=L+O$, add Z to both, then $Y+Z=L+O+Z$; but $Z+O=K$, put K inſtead of $Z+O$, then $Y+Z=L+K$, add to both Sides the equal Triangles B and A, then $Y+Z+B=L+K+A$. If therefore $B+Y+Z$ be taken from *abc*, and in lieu thereof we put $L+K+A$, we ſhall have the Figure *dfobikd*$=$*abc*, but that Figure is made up of the Meridian Diſtance when Eaſt, multiplied into the Southing, and the Meridian Diſtance, when Weſt, multiplied into the Northing, leſs by the Meridian Diſtance, when Weſt, multiplied into the Southing. Q. E. D.

COROLLARY.

Since the Meridian Diſtance (when Weſt) multiplied into the Southing, is to be ſubſtracted, by the ſame Reaſoning the Meridian Diſtance when Eaſt, multiplied into the Northing, muſt be alſo ſubſtracted.

SCHOLIUM.

From the two preceeding Theorems we learn how to find the Area of a Map, when the firſt Meridian paſſes through it; that is, when one Part of the Map lies on the Eaſt and the other on the Weſt Side of that Meridian. Thus,

RULE.

Rule.

The Merid. Dist. when $\left.\begin{array}{l}\text{East}\\ \text{West}\end{array}\right\}$ multiplied into the $\left\{\begin{array}{l}\text{Southings}\\ \text{Northings}\end{array}\right\}$; their Sum is th Area of the Map.

But,

The Merid. Dist. when. $\left\{\begin{array}{l}\text{East}\\ \text{West}\end{array}\right\}$ multiplied into the $\left\{\begin{array}{l}\text{Northings}\\ \text{Southings}\end{array}\right\}$ the Sum of these Products taken from the former, gives the Area of the Map.

These Theorems are true, when the Surveyor keeps the Land he surveys on his Right Hand, which we suppose thro' the whole to be done; but if he goes the contrary Way, call the Southings Northings, and the Northings Southings, and the same Rule will hold good.

From what has been demonstrated in Theo. 2. of this Section, it is plain, that if a Map lies to the East of the first Meridian, the Meridian Distances encrease when the Departures are East, and decrease when West. And again, if the Map lies to the West of the first Meridian, the Meridian Distances increase with West Departures, and decrease with East; from whence this general Rule for Meridian Distances may be inferred.

1. The Meridian Distance and Departure, both East, or both West, their Sum is the Meridian Distance of the same Name.

2. The Meridian Distance and Departure of different Names, that is, one East and the other West, their Difference is the Meridian Distance, of the same Name with the greater.

Thus in the third Method of finding the Area, as in the following Field-Book.

The first Departure is put opposite to the Northing or Southing of the first Station, and is the first Meridian Distance of the same Name. Thus if the first Departure be East, the first Meridian Distance will be the same as the Departure, and East also; and if West, it will be the same Way.

The first Meridian Distance	6.61	E
The next Departure	6.61	E
The second Meridian Distance	13.22	E
The next Departure	1.80	E
The third Meridian Distance	15.02	E

At Station 5, the Meridian Distance	5.78	E
The next Departure	7.76	W
The next Meridian Distance	1.98	W

At Station 11, the Meridian Distance	0.12	W
The next Departure	5.84	E
The next Meridian Distance	5.72	E

Plate X. Fig. 3.

In the 5th and 11th Stations the Meridian Distances being less than the Departures, and of a contrary Name, the Map will cross the first Meridian, and will pass as in the 5th Line, from the East to the

the West Side of the Meridian; and in the 11th Line it will again cross from the East to the West Side, which will evidently appear, if the Field-Work be protracted, and the Meridian Line passing through the first Station, be drawn through the Map.

The Field-Book cast up by the Third Method, will be evident from the two foregoing Theorems, and therefore requires no farther Explanation; but *to find the Area, by the* 4th *Method*, take this

RULE.

When the Meridian Distances are East, put the Products of North and South Areas in their proper Columns; but when West, in their contrary Columns; that is, in the Column of South Area, when the Difference of Latitude is North; and in North Area when South: The Reason of which is plain, from the two last Theorems. The Difference of these two Colums will be the Area of the Map.

By comparing the Area of the Field-Book by the two first and the two last Methods, it is plain, that the Column of Deductions of the two last, are much less than either of the two former, and consequently the Multiplications easier, as they consist of less Figures. Sometimes it will happen that there are no Deductions, but all the Area will stand in one Column, which cannot happen in either of the two former Methods.

The

No. Sta.	Bearings	C. L	Latit. and Half Dep.	Mer. Dist.	Area.	Deduction
1	NE 75	13.70	N 3.54 E 6.61	6.61 E 13.22 E		23.3994
2	NE $20\frac{1}{2}$	10.30	N 9.65 E+ 1.80	15.02 E 16.82 E		144.9430
3	East	16 20	0.00 E 8.10	24 92 E 33.02 E		
4	SW $33\frac{1}{2}$	35.30	S 29 44 W --- 9.74	23.29 E 13.54 E	685.3632	
5	SW 76	16.00	S 3 87 W 7.76	5.78 E 1.98 W	22.3686	
6	North	9 00	N 9 00 0.00	1.98 W 1.98 W	17.8200	
7	SW 84	11.60	S. 1.21 W---5.77	7.75 W 13.52 W		0.3775
8	NW $53\frac{1}{4}$	11.60	N 6 94 W---4.64	18.16 W 22.80 W	126.0304	
9	NE $36\frac{3}{4}$	19.20	N 15 38 E 5 74	17.06 W 11 32 W	262.3828	
10	NE $22\frac{1}{2}$	14.00	N 12.93 E 2.68	8.64 W 5 96 W	111.7152	
11	SE $76\frac{3}{4}$	12 00	S 2.75 E 5.84	0.12 W 5.72 E		0.3300
12	SW 15	10.85	S 10 48 W---1.40	4 32 E 2 92 E	45.2736	
13	SW $16\frac{3}{4}$	10.12	S 9.69 W---1.46	1.46 E 0.00	14.1474	
					1285.1012 178.0499	178.0499
	Content as before in Chains				1107.0513	

It is needleſs here to inſert the Columns of Bearing or Diſtances in Chains, being the ſame as before.

No. Sta.	Latit. and Half Dep.	Mer. Diſt.	N Area.	S Area.
1	N 3.54 E 6.61	6 61 E 13.22 E	23.3994	
2	N 9.65 E 1.80	15.02 E 16.82 E	144 9430	
3	0.00 E 8.10	24.92 E 33.02 E		
4	S 29.44 W 9.74	23.28 E 13.54 E		685.3632
5	S 3.87 W 7.76	5.78 E 1.98 W		22.3686
6	N 9.00 0.00	1.98 W 1.98 W		17.8200
7	S 1.21 W 5.77	7.75 W 13.52 W	9.3775	
8	N 6.94 W 4.64	18.16 W 22.80 W		126.0304
9	N 15.38 E 5.74	17 06 W 11.32 W		262.3828
10	N 12.93 E 2.68	8 64 W 5.96 W		111.7152
11	S 2.75 E 5.84	0.12 W 5.72 E	0.3300	
12	S 10.48 W 1.40	4.32 E 2.92 E		45.2736
13	S 9.69 W 1.46	1.46 E 0.00		14.1474
			178.0499	1285.1012
				178 0499
			Area in Chains as before	1107.0513

The Construction of the Map from either the 3d *or* 4th *Tables.*

Plate X. Fig. 3.

Draw the Line NS. for a North and South Line, which call the first Meridian; in this Line assume any Point, as 1, for the first Station. Set the Northing of that stationary Line which is 3.54 from 1 to 2, on the said Meridian Line. Upon the Point 2 raise a Perpendicular to the Eastward, the Meridian Distance being Easterly, and upon it set 13.22, the second Number in the Column of Meridian Distance, from 2 to 2, and draw the Line 1 2, for the first Distance Line: From 2 upon the first Meridian, set the Northing of the second stationary Line, that is, 9.65 to 3, and on the Point 3 erect a Perpendicular Eastward, upon which set the Meridian Distance of the second Station 16.82, from 3 to 3, and draw the Line 2 3, for the Distance Line of the second Station. And since the third Station has neither Northing nor Southing, set the Meridian Distance of it 33.02, from 3 to 4, for the Distance Line of the third Station. To the fourth Station there is 29.44, southing which set from 3 to 5; upon the Point 5 erect the Perpendicular 5 5, on which lay 13.54, and draw the Line 4 5.

In the like Manner proceed to set the Northings and Southings on the first Meridian, and the Meridian Distances upon the Perpendiculars raised to the East or West; the Extremities of which connected by Right Lines, will compleat the Map.

A Map may be in like Manner constructed from the first or second Tables.

A TABLE.

A TABLE *of Difference of Latitude and Half Departure, or of Northing, Southing, Half Easting, and Half Westing, to every Quarter Degree of the Compass; the Distance being supposed to be One Four-Pole Chain.*

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. E.	E. W.
0.		1.0000	.0000	6.	0	.9945	.0522
	1	1.0000	.0022		1	.9940	.0544
	2	1.0000	.0044		2	.9936	.0566
	3	.9999	.0065		3	.9931	.0587
1.	0	.9998	.0087	7.	0	.9925	.0609
	1	.9997	.0109		1	.9920	.0631
	2	.9996	.0131		2	.9914	.0652
	3	.9995	.0153		3	.9909	.0674
2.	0	.9994	.0175	8.	0	.9903	.0696
	1	.9992	.0196		1	.9896	.0717
	2	.9990	.0218		2	.9890	.0739
	3	.9988	.0240		3	.9883	.0760
3.	0	.9986	.0262	9.	0	.9877	0782
	1	.9984	.0283		1	.9870	.0804
	2	.9981	.0305		2	.9863	.0825
	3	.9978	.0327		3	.9855	.0847
4.	0	.9975	.0349	10.	0	.9848	.0868
	1	.9972	.0370		1	.9840	.0889
	2	.9969	.0392		2	.9832	.0911
	3	.9966	0414		3	.9824	.0933
5.	0	.9662	.0436	11.	0	.9816	.0954
	1	.9958	.0457		1	.9808	.0975
	2	.9954	.0479		2	.9799	.0997
	3	.9949	.0501		3	.9790	.1018

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
12.	0	.9781	.1039	21.	0	.9336	.1792
	1	.9772	.1061		1	.9320	.1812
	2	.9763	.1082		2	.9304	.1832
	3	.9753	.1103		3	.9288	.1852
13.	0	.9744	.1124	22.	0	.9272	.1873
	1	.9734	.1146		1	9255	.1893
	2	.9724	.1167		2	.9239	.1913
	3	.9713	.1188		3	.9222	.1933
14.	0	.9703	.1209	23.	0	.9205	.1953
	1	.9692	.1231		1	9188	.1973
	2	.9681	.1252		2	.9171	.1993
	3	.9670	.1273		3	9153	.2013
15.	0	.9659	.1294	24.	0	.9135	.2033
	1	.9647	.1315		1	.9117	.2053
	2	.9636	.1336		2	.9099	.2073
	3	.9624	.1357		3	.9081	.2093
16.	0	.9613	.1378	25.	0	.9063	.2113
	1	.9600	.1399		1	.9044	.2133
	2	.9588	.1420		2	.9026	.2152
	3	.9576	.1441		3	.9007	.2172
17.	0	.9563	.1462	26.	0	.8988	.2192
	1	.9550	.1482		1	.8968	.2211
	2	.9537	.1503		2	.8949	2230
	3	.9524	.1524		3	.8929	2250
18.	0	.9510	.1545	27.	0	.8910	.2270
	1	.9497	.1566		1	.8890	.2289
	2	.9483	.1586		2	.8870	.2308
	3	.9469	.1607		3	.8850	.2328
19.	0	.9455	.1627	28.	0	.8829	.2347
	1	.9441	.1648		1	.8809	.2366
	2	.9426	.1669		2	.8788	.2385
	3	.9412	.1689		3	.8767	2405
20.	0	.9397	.1710	29.	0	.8746	.2424
	1	.9382	.1730		1	.8725	2443
	2	.9366	.1751		2	.8703	.2462
	3	.9351	.1771		3	.8682	.2481

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
30.	0	.8660	.2500	39.	0	.7771	.3146
	1	.8638	.2519		1	.7744	.3163
	2	.8616	.2537		2	.7716	.3180
	3	.8594	.2556		3	.7688	.3197
31	0	.8571	.2575	40.	0	.7660	.3214
	1	.8549	.2594		1	.7632	.3230
	2	.8527	.2612		2	.7604	.3247
	3	.8503	.2631		3	.7575	.3263
32.	0	.8480	.2649	41.	0	.7547	.3280
	1	.8457	.2668		1	.7518	.3296
	2	.8434	.2686		2	.7489	.3313
	3	.8410	.2705		3	.7460	.3329
33.	0	.8387	.2723	42.	0	.7431	.3345
	1	.8363	.2741		1	.7402	.3361
	2	.8339	.2759		2	.7373	.3378
	3	.8315	.2778		3	.7343	.3394
34.	0	.8290	.2796	43.	0	.7313	.3410
	1	.8266	.2814		1	.7282	.3426
	2	.8241	.2832		2	.7254	.3441
	3	.8216	.2850		3	.7224	.3457
35.	0	.8191	.2867	44.	0	.7192	.3473
	1	.8166	.2885		1	.7163	.3489
	2	.8141	.2903		2	.7132	.3504
	3	.8116	.2921		3	.7102	.3520
36.	0	.8090	.2939	45.	0	.7071	.3535
	1	.8064	.2956		1	.7041	.3551
	2	.8038	.2974		2	.7009	.3566
	3	.8012	.2991		3	.6978	.3581
37.	0	.7986	.3009	46.	0	.6946	.3596
	1	.7960	.3026		1	.6915	.3612
	2	.7933	.3044		2	.6883	.3627
	3	.7907	.3061		3	.6852	.3642
38.	0	.7880	.3078	47.	0	.6820	.3656
	1	.7853	.3095		1	.6788	.3671
	2	.7826	.3112		2	.6756	.3686
	3	.7799	.3129		3	.6724	.3701

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	N. S.	E. W.
48.	0	.6691	.3715	57.	0	.5446	.4193
	1	.6659	.3730		1	.5408	.4205
	2	.6526	.3745		2	.5373	.4217
	3	.6593	.3759		3	.5336	.4228
49.	0	.6560	3773	58	0	.5299	.4240
	1	.0527	3788		1	.5262	.4251
	2	.6494	.3802		2	5225	.4263
	3	.6401	.3816		3	.5188	.4274
50.	0	.6428	.3830	59.	0	.5150	.4286
	1	.6394	.3844		1	.5113	.4297
	2	.6361	.3858		2	5075	.4308
	3	.6327	.3872		3	5038	.4319
51.	0	.6293	.3885	60.	0	.5000	.4330
	1	.6259	.3899		1	.4962	.4341
	2	.6225	3913		2	.4924	.4351
	3	6191	.3926		3	.4886	.4362
52.	0	.6157	.3940	61.	0	.4848	.4373
	1	.6122	.3953		1	.4810	.4383
	2	.6087	.3966		2	.4771	.4394
	3	.6052	.3980		3	.4733	.4404
53.	0	.6018	.3993	62	0	.4695	.4414
	1	.5983	.4006		1	.4656	.4425
	2	.5948	.4019		2	.4617	.4435
	3	.5912	.4032		3	.4579	.4445
54.	0	.5878	.4045	63	0	.4540	.4455
	1	.5842	.4058		1	.4501	.4464
	2	.5807	.4070		2	.4462	.4474
	3	.5771	.4083		3	.4423	.4484
55.	0	.5736	.4095	64.	0	.4383	.4494
	1	.5700	.4108		1	4344	.4503
	2	.5664	.4120		2	.4305	.4513
	3	.5628	.4133		3	.4265	.4522
56.	0	.5592	.4145	65.	0	.4226	.4531
	1	.5555	.4157		1	.4186	.4540
	2	.5519	.4169		2	.4147	.4549
	3	.5483	.4181		3	.4107	.4558

Deg.	Qrs	N. S.	E. W.	Deg.	Qrs.	N. S.	W. E.
66.	0	.4067	.4567	75	0	.2588	.4829
	1	.4027	.4576		1	.2546	.4835
	2	.3987	.4585		2	.2504	.4840
	3	.3947	.4594		3	.2461	.4846
67.	0	.3907	.4602	76	0	.2419	.4851
	1	.3867	.4611		1	.2377	.4856
	2	.3827	.4619		2	.2334	.4862
	3	.3786	.4627		3	.2292	.4867
68.	0	.3746	.4636	77.	0	.2249	.4872
	1	.3705	.4644		1	.2207	.4876
	2	.3665	.4652		2	.2164	.4881
	3	.3624	.4660		3	.2122	.4886
69.	0	.3584	.4668	78.	0	.2079	.4890
	1	.3543	.4676		1	.2036	.4895
	2	.3502	.4683		2	.1993	.4899
	3	.3461	.4691		3	.1951	.4904
70.	0	.3420	.4698	79.	0	.1908	.4908
	1	.3379	.4706		1	.1865	.4912
	2	.3338	.4713		2	.1822	.4916
	3	.3297	.4720		3	.1779	.4920
71.	0	.3255	.4727	80	0	.1736	.4924
	1	.3214	.4734		1	.1693	.4927
	2	.3173	.4741		2	.1650	.4931
	3	.3131	.4748		3	.1607	.4935
72.	0	.3090	.4755	81.	0	.1564	.4938
	1	.3048	.4762		1	.1521	.4941
	2	.3007	.4768		2	.1478	.4945
	3	.2965	.4775		3	.1435	.4948
73.	0	.2924	.4781	82.	0	.1392	.4951
	1	.2882	.4788		1	.1348	.4954
	2	.2840	.4794		2	.1305	.4957
	3	.2798	.4800		3	.1262	.4960
74.	0	.2756	.4806	83.	0	.1218	.4962
	1	.2714	.4812		1	.117[illegible]	.4965
	2	.2672	.4818		2	.113[illegible]	.4968
	3	.2630	.4823		3	.10[illegible]	.4970

Deg.	Qrs.	N. S.	E. W.	Deg.	Qrs.	S. N.	E. W.
84.	0	.1045	.4972	87.	0	.0523	.4993
	1	.1002	.4974		1	.0480	.4994
	2	.0958	.4977		2	.0436	.4995
	3	.0915	.4979		3	.0392	.4996
85.	0	.0871	.4981	88.	0	.0349	.4997
	1	.0828	.4983		1	.0305	.4997
	2	.0784	.4984		2	.0262	.4998
	3	.0741	.4986		3	.0218	.4998
86.	0	.0697	.4988	89.	0	.0174	.4999
	1	.0654	.4989		1	.0131	.4999
	2	.0610	.4990		2	.0087	.5000
	3	.0566	.4992		3	.0043	.5000
				90.	0	.0000	.5000

A TABLE

A TABLE *for changing Degrees taken by the Circumferentor to those of the Quartered Compass, and the contrary.*

Degrees		Degrees		Degrees		Degrees	
Cir.	Q.C.	Cir.	Q.C.	Cir	Q.C.	Cir.	Q.C.
360	Nor.		NW		NW.		S.W.
1	NW. 1	31	31	61	61	91	89
2	2	32	32	62	62	92	88
3	3	33	33	63	63	93	87
4	4	34	34	64	64	94	86
5	5	35	35	65	65	95	85
6	6	36	36	66	66	96	84
7	7	37	37	67	67	97	83
8	8	38	38	68	68	98	82
9	9	39	39	69	69	99	81
10	10	40	40	70	70	100	80
11	11	41	41	71	71	101	79
12	12	42	42	72	72	102	78
13	13	43	43	73	73	103	77
14	14	44	44	74	74	104	76
15	15	45	45	75	75	105	75
16	16	46	46	76	76	106	74
17	17	47	47	77	77	107	73
18	18	48	48	78	78	108	72
19	19	49	49	79	79	109	71
20	20	50	50	80	80	110	70
21	21	51	51	81	81	111	69
22	22	52	52	82	82	112	68
23	23	53	53	83	83	113	67
24	24	54	54	84	84	114	66
25	25	55	55	85	85	115	65
26	26	56	56	86	86	116	64
27	27	57	57	87	87	117	63
28	28	58	58	88	88	118	62
29	29	59	59	89	89	119	61
30	30	60	60	90	90	120	60

Degrees		Degrees		Degrees		Degrees	
Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C.
	S.W.		S.W.		S.E.		S.E.
121	59	151	29	181	1	211	31
122	58	152	28	182	2	212	32
123	57	153	27	183	3	213	33
124	56	154	26	184	4	214	34
125	55	155	25	185	5	215	35
126	54	156	24	186	6	216	36
127	53	157	23	187	7	217	37
128	52	158	22	188	8	218	38
129	51	159	21	189	9	219	39
130	50	160	20	190	10	220	40
131	49	161	19	191	11	221	41
132	48	162	18	192	12	222	42
133	47	163	17	193	13	223	43
134	46	164	16	194	14	224	44
135	45	165	15	195	15	225	45
136	44	166	14	196	16	226	46
137	43	167	13	197	17	227	47
138	42	168	12	198	18	228	48
139	41	169	11	199	19	229	49
140	40	170	10	200	20	230	50
141	39	171	9	201	21	231	51
142	38	172	8	202	22	232	52
143	37	173	7	203	23	233	53
144	36	174	6	204	24	234	54
145	35	175	5	205	25	235	55
146	34	176	4	206	26	236	56
147	33	177	3	207	27	237	57
148	32	178	2	208	28	238	58
149	31	179	1	209	29	239	59
150	30	180	Sou.	210	30	240	60

Degrees		Degrees		Degrees		Degrees	
Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C.	Cir.	Q.C.
	S. E.		S. W.		N. E.		N. E
241	61	271	89	301	59	331	20
242	62	272	88	302	58	332	28
243	63	273	87	303	57	333	27
244	64	274	86	304	56	334	26
245	65	275	85	305	55	335	25
246	66	276	84	306	54	336	24
247	67	277	83	307	53	337	23
248	68	278	82	308	52	338	22
249	69	279	81	309	51	339	21
250	70	280	80	310	50	340	20
251	71	281	79	311	49	341	19
252	72	282	78	312	48	342	18
253	73	283	77	313	47	343	17
254	74	284	76	314	46	344	16
255	75	285	75	315	45	345	15
256	76	286	74	316	44	346	14
257	77	287	73	317	43	347	13
258	78	288	72	318	42	348	12
259	79	289	71	319	41	349	11
260	80	290	70	320	40	350	10
261	81	291	69	321	39	351	9
262	82	292	68	322	38	352	8
263	83	293	67	323	37	353	7
264	84	294	66	324	36	354	6
265	85	295	65	325	35	355	5
266	86	296	64	326	34	356	4
267	87	297	63	327	33	357	3
268	88	298	62	328	32	358	2
269	89	299	61	329	31	359	1
270	East	300	60	330	30	360	Nor.

The above Table is ſo plain in itſelf, that it needs no further Explanation than this, that Cir. at the Top of the Column ſtands for Circumferentor, and Q. C. for Quarter'd Compaſs.

 To

To change Degrees of the Circumferentor to those of the Quarter'd Compass, without the foregoing Table, and the contrary.

1. If the Degrees are less than 90, they are that Number in the NW Quadrant.

2. If the Degrees are between 90 and 180, take them from 180, and the Remainder is the Degrees in the SW Quadrant.

3. If the Degrees are between 180 and 270, take 180 therefrom, and the Residue is the Degrees in the SE Quadrant.

4. If the Degrees are between 270 and 360, take them from 360, and the Remainder is the Degrees in the NE Quadrant.

5. 360 is N, 180 is S, 90 is W, and 270 is E.

EXAMPLE.

1. Change $56^{\circ}\frac{1}{2}$ taken by a Circumferentor, to Degrees of the Quarter'd Compass.

Answer N W $56\frac{1}{2}$

2. Change $126\frac{1}{4}$ to Degrees of the Quarter'd Compass.

$$\begin{array}{lr} & 180 \\ & 126\frac{1}{4} \\ \hline \text{Answer} & 53\frac{3}{4} \\ \hline \end{array}$$

3. Change

3. Change $234^{\circ}\frac{1}{4}$ to Degrees of the Quarter'd Comp.

$$\begin{array}{r} 234\frac{1}{4} \\ 180 \\ \hline 54\frac{1}{4} \\ \hline \end{array}$$

Anſwer SE $54\frac{1}{4}$

4. Change $324^{\circ}\frac{3}{4}$ to Degrees of the Quarter'd Compaſs.

$$\begin{array}{r} 360 \\ 324\frac{3}{4} \\ \hline 35\frac{1}{4} \end{array}$$

Anſwer NE $35\frac{1}{4}$

The contrary of the former.

Again to change Degrees taken by the Quarter'd Compaſs to thoſe of the Circumferentor.

1. If the Degrees are in the NW Quadrant, they are a like Number by the Circumferentor.

2. If the Degrees are in the SW Quadrant, take them from 180, for the Degrees by the Circumferentor.

3. If the Degrees are in the SE Quadrant, add them to 180, their Sum will be thoſe for the Circumferentor.

4. If the Degrees are in the NE Quadrant, take them from 360, the Remainder will be thoſe for the Circumferentor.

5. N is 360, S 180, W 90, and E 270 Degrees.

This is ſo plain it requires no Example.

Having the Angles of the Field taken by a Theodolite, Semicircle, or Plane Table given, to reduce them to Angles from the Meridian, or to those taken by the Circumferentor.

THE Angles of the Field, with the Bearing of the first, or any other stationary Line from the Meridian being given; the Bearings of all the other stationary Lines may be found by these two Rules.

Rule I.

If the Angle of the Field at any Station be more than 180, take 180 from it, and add the Remainder to the Bearing at the foregoing Station; the Sum if less than 360 will be the Bearing at the present Station or of the next Line: But if the Sum be more than 360, take 360 from it, and the Remainder will be the present Bearing.

Rule II.

If the Angle of the Field be less than 180 take it from 180, and from the Bearing at the foregoing Station take the Remainder, and you will have the Bearing of the present one: But if the Bearing at the foregoing Station be less than the first Remainder, to this foregoing Bearing add 360, and from that Sum take the first Remainder, and this last Remainder will be the present Bearing. An Example will render this Matter familiar and easy.

Required to change the following Angles of the Field, to Bearings of the Needle from the North, the Bearing of the first stationary Line being 262.

The

No. Sta.	Ang. Field	The Manner of finding the Bearings.	Bearings.	No. Sta.
1	159			
2	200	200—180=20, 262+20=	282	2
3	270	270—180=90, 282+90=372, 372—360=	12	3
4	80	180—80=100, 12+360=372, 372—100=	272	4
5	98	180—98=82, 272—82=	190	5
6	100	180—100=80, 190—80=	110	6
7	230	230—180=50, 50+110=	160	7
8	90	180—90=90, 160—90=	70	8
9	82	180—82=98, (70—82)=360+70—98=	332	9
10	191	191—180=11, 11+332=	343	10
11	120	180—120=60, 343—60=	283	11
		180—159=21, 283—21=	262	1
	1620	Sum of the Angles in the Field		

The last found Bearing being the same as that taken at the first Station by the Needle, shews that all the Angles of the Field were truly taken.

The like we may also be confirmed of by adding up the Angles of the Field; for if their Sum be equal to twice as many right Angles as there are Stations or Sides in the Figure less by 4, they are truly taken. By Scholium, P. 157.

22 twice the Number of Stations
4

18 Right Angles
90

1620 Sum of the Angles

Having found the Bearings from the North, or by the Circumferentor, the Bearings by the Quarter'd Compass are easily obtained as before. And thus

thus a Map can be more accurately protracted than from the Angles of the Field, and its Area may be had by Calculation.

If you have all the Angles of the Field without any Bearing given, you may in the like Manner reduce them to Bearings, which will anſwer for Protraction by Parallels, by finding the Area by Calculation; by ſuppoſing the Bearing of the firſt ſtationary Line to be any Number leſs than 360, and thence you may alſo know, if the Angles of the Field be truly taken. For though the Bearings of the ſeveral ſtationary Lines be falſe ones, yet the Map will be equal in all Reſpects to one protracted from Bearings taken by the Needle, and therefore the Area of each will be the ſame: But you will be at a loſs for the Situation of the Land ſurveyed, with reſpect to the North, South, Eaſt, and Weſt Points of the Compaſs.

The foregoing Rules demonſtrated.

Plate XI. Fig. 1.

Let 1, 2, 3, 4, 5, &c. repreſent a Plot of Ground, and let the Lines NS paſſing through every Station repreſent Meridian Lines.

The Arc NSo at the firſt Station, will be the Bearing of the Line 1 2, from the North, as if taken by the Circumferentor, which Arc is equal to the Arc Nr Sq at the ſecond Station, made by producing the Line 1 2, from the Parallel Poſition of the Needle. And the Arc rSqo at the ſecond Station,—rSq which is a Semicircle, is$=$oq, but NrSq, which was ſhewn to be equal to the Bearing at the firſt Station$+$oq$=$NrSqo, which will be the Bearing of the ſecond ſtationary Line.

Thus

Thus if from rSqo 200°, the Angle of the Field at the ſecond Station, rSq = 180° be taken, the Remainder oq will be 20°. But NrSq = 262° the Bearing of the firſt ſtationary Line†oq, which is 20° = NrSoq = 282°, the Bearing of the ſecond ſtationary Line.

Again at the third Station NrSq = NrSqo, the Bearing of the ſecond ſtationary Line, and rSqNo the Angle of the Field at the third Station; if then from it the Semicircle rSq be taken, the Remainder will be qNo, to which if NrSqo be added, the Sum will be the entire Circle † No, therefore if the Circle, or 360, be taken therefrom, the Remainder No will be the Bearing of the third ſtationary Line, *&c.* Q. E. D.

All the reſt will be manifeſt with little Conſideration when every Line is continued, and the Circles drawn as in the Scheme.

SECT.

SECT. VI.

Containing the Nature of Off-ſets and Interſections; the Methods of enlarging, diminiſhing, and connecting Maps, with the Method of tracing the Down (or any other) Surveys. Alſo the Variation of the Compaſs and its Uſes in Surveying. How to reduce one Meaſure to another in ſix Problems, with three other very uſeful Problems; and the whole concludes with ſome neceſſary Derections concerning Surveys in general.

Of OFF-SETS.

IN taking Surveys it is unneceſſary and unuſual to make a Station at every angular Point, becauſe the Field Work can be taken with much greater Expedition, by uſing Off-ſets and Interſections, and with equal Certainty.

Off-ſets are perpendicular Lines drawn or meaſured from the angular Points of the Land, that lie on the right or left Hand to the ſtationary Diſtance, thus,

Plate XI. Fig. 2.

Let the black Lines repreſent the Mearings of a Farm or Town-Land: And let 1 be the firſt Station

tion, then if you have a good View to 2, you omit the angular Points between 1 and 2, and take the Bearing and Length of the ſtationary Line 1 2, and inſert them in your Field-Book: But in chaining from 1 to 2, ſtop at *d* oppoſite to the angular Point a, and in your Field-Book inſert the Diſtance from 1 to d, which admit to be 4C. 25L. as well as the Meaſure of the Off-ſet ad, which admit to be 1C. 12L. thus: By the Side of your Field-Book in a Line with the firſt Station ſay, at 4C. 25L. 1C. 12L. that is, at 4C. 25L. there is an Off-ſet to the left Hand of 1C. 12L.

This done, proceed on your diſtance Line to e, oppoſite to the Angle b, and meaſure eb, ſuppoſing then 1e to be 7C. 40L. and eb 3C. 40L. Say (ſtill in a Line with the firſt Station in your Field-Book) at 7C. 40L. L3C. 40L. that is at 7C. 40L. there is an Off-ſet to the left of 3C. 40L. proceed then with your diſtance Line to f, oppoſite to the Angle c, and meaſure fc; ſuppoſe then 1f to be 13C. and fc 1C. 25L. ſay in the ſame Line as before, at 13C. L1C. 25L. Then proceed from f to 2, and you will have the Meaſure of the entire ſtationary Line 1 2, which inſert in its proper Column by the Bearing.

In taking Off-ſets, it is neceſſary to have a Perch Chain, or a Staff of half a Perch, divided into Links for meaſuring them; for by theſe Means the Chain in the ſtationary Line is undiſturbed, and the Number of Chains and Links in that Line from whence, or to which, the Off-ſets are taken may be readily known.

Having arrived at the ſecond Station if you find your View will carry you to 3, take the Bearing from 2 to 3, and in meaſuring the Diſtance Line,

ſtop at l oppoſite to g; admit 2l to be 4C. 10L. and the Off-ſet lg 1C. 20L. then in a Line with the ſecond Station in your Field-Book, ſay at 4C. 10L. R 1C. 20L. that is the Off-ſet is a right Hand one of 1C. 20L. Again at m, which ſuppoſe to be 10C. 25L. from 2; take the Off-ſet mh of 1C. 15L. and in a Line with the ſecond Station ſay, at 10C. 25L. R 1C. 15L. In the ſame Line when you come to the Mearing at *i*, inſert the Diſtance 2*i*, 13C. 10L. thus, at 13C. 10L.0; that is at 13C. 10L. there is one Off-ſet. At *n*, which is 15C. from 2, take the Off-ſet *nk* 45L. and ſtill oppoſite to the ſecond Station ſay at 15C. L45L.

Let the Line 36 repreſent the Mearing, which by Means of Water, Bryars, or any other Impediment cannot be meaſured. In this Caſe make one or more Stations within or without the Land, where the Diſtances may be meaſured, and draw a Line from the Beginning of the firſt to the End of the laſt Diſtance thus: Make Stations at 3, 4, and 5, taking the Bearings, and meaſuring the Diſtances as uſual, which inſert in your Field-Book, and draw a Mark like one Side of a Parentheſis from the third to the fifth Station, to ſhew that a Line drawn from the third Station to the fartheſt End of the fifth ſtationary Line will expreſs the Mearing. Thus,

No. Sta.	Deg.	C. L,
(3	$172\frac{1}{2}$	5.45
4	200	13.25
5	250	3.36

Suppoſe the Point *p* of the Mearing to be inacceſſible, by Means of the Lines 6*p* or *p*7, being overflowed, or that a Quarry, Furze, &c. might prevent you taking their Lengths: In this Caſe take the Bearing of the Line 67, which inſert op-

poſite

poſite to the ſixth Station in your Field-Book with the other Bearings; then direct the Index to the Point *p*, and inſert its Bearing on the left Side of the Field-Book, oppoſite to the ſixth Station, annexing thereto the Words, *Int. for Mearing*, and having meaſured and inſerted the Diſtance 6 7, ſet the Index in the Direction of the Line 7*p*, and inſert its Bearing on the left of the ſeventh Station of the Field-Book, annexing thereto the Words *Int. for Mearing*; the Croſſing or Interſection of theſe two Bearings will determine the Point *p*, and of courſe the Mearing 6*p*7 is alſo determined.

If your View will then reach to the firſt Station, take its Bearing, ſtationary Line, and Off-ſets, as before, and you have the Field-Book compleated. Thus,

The FIELD-BOOK.

Remarks and Interſect.	No. Sta.	Deg.	C. L.	OFF-SETTS.
318 Int. to a Tower	1	358	22.12	*At* 4*C* 25*L*. *L* 1*C*. 12*L*. *at* 7*C*. 40*L*. *L* 3*C*. 40*L*. *at* 13*C*. *L* 1*C*. 25*L*.
231½ Int. to Ditto.	2	297¾	21.12	*At* 4*C*. 10*L*. *R* 1*C*. 20*L*. *at* 10*C*. 25*L*. *R* 1*C*. 15*L*. *at* 13*C*. 10*L*. o. *at* 15*C*. *L*. 45*L*.
	3	172¼	5.45	
	4	200	13.25	
	5	250	3.36	
155½ Int. for Mearing	6	125	15.15	
274 Int. for Mearing	7	105¼	15.10	*At* 1*C*. 20*L*. *L* 2*C*. 20*L* : *at* 7*C*. 45*L* *L* 2*C*. 32*L* *at* 11*C*. 25*L*. o *at* 12*C*. 25*L*. *R* 36*L*.

Cloſe at the firſt Station.

If you would lay down a Tower, Houſe, or any other remarkable Object in its proper Place; from

any two Stations take Bearings to the Object, and their Intersection will determine the Place where you are to insert it, in the Manner that the Tower is set out in the Figure, from the Intersections taken at the first and second Stations of the above Field-Book.

A Protraction of this will render all plain, on which lay off your Off-sets and Intersections, and proceed to find the Content by any of the Methods in Section the 4th.

The foregoing Field-Book may be otherwise kept thus:

Remarks and Intersection	No. Stat	Deg.	L-Han. Off-set C. L.	Dif. C. L.	R. Han. Off-set C. L.
318 Int. to a Tower	1	358	1.12	4.25	
			3.40	7.40	
			1.25	13.00	
				22.12	
231½ Int. for Ditto	2	297¾		4.10	1.20
				10.25	1.15
				13.10	
			0.45	15.00	
				21.12	
	3	172½		5.45	
	4	200		13.25	
	5	250		3.36	
155½ Int. for Mearings	6	125		15.15	
274 Int. for Mearing	7	105	2.20	1.20	
			2.32	7.45	
				11.25	
				12.25	0.30
				15.10	

Close at the first Station.

How

How to cast up Off-sets by the Pen.

Plate XI. Fig. 2.

$12-1f=2f$, $1f-1e=fe$, $1e-1d=ed$.

Then $1d \times \frac{1}{2}da=1da$, by Prob. 6, Page 183, and $\frac{1}{2}\ ed \times da+eb=adeb$ by Lemma, Page 226, also $\frac{1}{2}\ fe \times eb+fc=befc$, and $2f \times \frac{1}{2}\ fc=cf2$; the Sum of all which will be $1abc21$; the Area contained between the stationary Line 12, and the Mearing, 1 *abc* 2.

In the same Manner you may find the Area of $2ihg2$ of $ik3i$, as well as what is without and withinside of the Stationary Line 71.

If therefore the left Hand Off-sets exceed the right Hand ones, it is plain, the Excess must be added to the Area within the stationary Lines, but if the right Hand Off-sets exceed the left-Hand ones, the Difference must be deducted from the said Area; if the Ground be kept on the right Hand as we have all along supposed; or in Words thus.

To find the Contents of Off-sets.

1. From the Distance Line, take the Distance to the preceeding Off-set, and from that the Distance of the one preceeding it, *&c.* in Four-Pole Chains; so will you have the respective Distances from Off-set to Off-set, but in a retrograde Order.

2. Multiply the last of these Remainders by ½ the first Off-set, the next by ½ the Sum of the first

first and second, the next by half the Sum of the second and third, the next by half the Sum of the third and fourth, &c. The Sum of these will be the Area produced by the Off-sets.

Thus, in the foregoing Field-Book, the first stationary Line is 22C. 12L. or 11C. 12L. of 4 Pole Chains. See the Figure.

	C. L.	C. L	C. L.
From	11.12=1,2	6.50=1*f*	3.90=1*e*
Take	6.50=1*f*	3.90=1*e*	2.25=1*d*
	4.62=2*f*	2.60=*ef*	1.65=*ed*

C. L.

1*d*=2.25 x 32L. half the first Off-set=	.7200
ed= 1.65 x 1C. 26L. ½ the Sum of the 1st&2d	2.0790
ef= 2.60 x 1C. 32L. ½ the Sum of 2d & 3d=	3.4320
2*f*= 4.62 x 37L. half the last Off-set=	1.7094
Content of left Off-sets on the first Dist. in Square Four-pole Chains	7.9404

In like Manner the rest are perform'd.

The Sum of the left Hand Off-set will be	14.0856
And the Sum of the right Hand ones	3.6825
Excess of left Hand Off-sets in Squ. 4PoleC.	10.4031
Acres	1.04031
	.16124
	4
Perch	6.4496

Excess of left Hand Off-sets above the right Hand ones, 1A. 0R. 6P. to be added to the Area within the stationary Lines.

Of

Of INTERSECTIONS.

How to find the Area of a Piece of Ground by Interſections only, when all the Angles of the Field can be ſeen from any two Stations on the out Side of the Ground.

Plate XII. Fig. 1.

LET ABCDEFGA be a Field, H and I two Places on the out Side of it, from whence an Object at every Angle of the Field may be ſeen.

Take the Bearing and Diſtance between H and I, and ſet that at the Head of your Field-Book, as in the annexed one. Fix your Inſtrument at H, from whence take the Bearings of the ſeveral angular Points, A, B, C, D, *&c.* as they are here repreſented by the Lines HA, HB, HC, HD, *&c.* Again fix your Inſtrument at I, and take Bearings to the ſame angular Points, repreſented by the Lines IA, IB, IC, ID, *&c.* and let the firſt Bearings be entered in the ſecond Column, and the ſecond Bearings in the third Column of your Field-Book: Then it is plain that the Points of Interſection, made from the Bearings in the ſecond and third Columns of every Line, will be the Angular Points of the Field or the Points A, B, C, D, *&c.* which Points being joined by right Lines, will give the Plan ABCDEFGA required.

Bea. 180 Dis. 28C. of the Sta. H and I.		
No.	Bear.	Bear.
A	261½	331½
B	265¾	317¼
C	248	307½
D	238¼	289
E	215½	262½
F	208½	286½
G	220	300

The ſame may be done from any two Stations within-ſide of the Land, from whence all the Angles of the Field can be ſeen.

This Method will be found uſeful in Caſe the ſtationary Diſtances from any Cauſe prove inacceſſible, or ſhould it be required to be done by one Party, when the other whoſe Poſſeſſion it is, refuſes to admit you to go on the Land.

To find the Content of a Field by Calculation, which was taken by Interſection.

In the Triangle AIH, the Angles AHI, AIH, and the Baſe HI being known, the Perpendicular A*a*, and the Segments, of the Baſe *Ha*, *AI* may be obtained by Trigonometry: And in the ſame Manner all the other Perpendiculars B*b*, C*c*, D*d*, E*e* F*f*, G*g*, and the ſeveral Segments at *b*, *c*, *d*, *e*, *f*, and *g*: If therefore the ſeveral Perpendiculars be ſuppoſed to be drawn in the Scheme, (which are here omitted to prevent Confuſion ariſing from a Multiplicity of Lines) it is plain that if from

*b*BCDE*eb*, there be taken *b*BAGF*eb*, the Remainder will be the Map ABCDEFGA.

As before, Half the Sum of B*b*, and C*c*, multiplied by *bc*, will be the Area of the Trapezium *b*BC*c*; after the ſame Manner, Half the Sum of C*c*, and D*d*, multiplied by *cd*, will give the Area of the Trapezium *c*CD*d*; and again, Half the Sum of D*d*, and E*e* multiplied by *de*, gives the Area of the Trapezium *d*DE*e*; and the Sum of theſe three Trapezia will be the Area of the Figure *b*BCDE*eb*.

Again, in the ſame Manner, Half the Sum of *b*B, and *aA* multiplied by *ab*, will give the Area of the Trapezium *b*BA*a*; and Half the Sum of *a*A, and *g*G, by *ag*, gives the Trapezium *a*AG*g*; to theſe add the Trapezia *g*GF*f*, and *f*FE*e*, which are found in the like Manner, and you will have the Figure *b*BAGFE*eb*, and this taken from *b*BCDE*eb*, will leave the Map ABCDEFGA. Q. E. F.

It will be ſufficient to protract this Kind of Work, and from the Map to determine the Area, as well as in Plate X Fig. 3. to find the Areas of the Pieces 34563, and 6*p*76, from Geometrical Conſtructions.

How to determine the Station where a Fault has been committed in a Field-Book, without the Trouble of going round the whole Ground a ſecond Time.

From every fourth or fifth Station, if they be not very long ones, or oftner if they are, let an Interſection be taken to any Object, as to any particular

Part of a Caſtle, Houſe, or Cock of Hay, *&c.* or if all theſe be wanting, to a long Staff with a white Sheet or Napkin ſet thereon to render the Object more conſpicuous, and let this be placed on the Summit of the Land, and let the reſpective Interſections ſo taken be inſerted on the left Hand Side of the Field-Book, oppoſite to the Stations from whence they were reſpectively taken.

In your Protraction as you proceed, let every Interſection be laid off from the reſpective Stations from whence they were taken, and let theſe Lines be continued; if they all converge or meet in one Point we thence conclude all is right, or ſo far as they do converge; but if we find a Line of Interſection to diverge or fly off from the reſt, we may be ſure that either a Miſtake has happened between the Station, the foregoing Interſection was taken at, and the Station from whence the Interſection Line diverges; or there muſt be an Error in the Interſection; but to be aſſured in which of theſe the Fault is, protract on to the next Interſection, and having ſet it off, if it converges with the reſt, tho' the foregoing one did not, we may conclude the Fault was committed in taking the laſt Interſection but one, and none in any Station, and that ſo far is true as is protracted; but if this as well as the foregoing Interſection diverge, or fly from the Point of Concourſe or converging Point of the reſt, the Error muſt have its Riſe from ſome Station or Stations, at or after that, from whence the laſt converging Interſection Line was taken; ſo that by going to that Station on the Ground and proceeding on to that where the next, or from whence the following diverging Interſection was taken we can readily and with little Trouble ſet all to Rights.

But in moſt Tracts of Land, one Object cannot be ſeen from every Station, or from perhaps one fourth of them; in this Caſe we are under the Neceſſity to move the Pole after we begin to loſe Sight of it to ſome other Part of the Land, where it may be ſeen from as many more Stations as poſſible; which is eaſily done by viewing the Mearing before it be ſurveyed: The Pole then being fixed in an advantageous Place, the firſt Interſection to it is beſt to be made from the ſame Station from whence the laſt one was taken, and then as often as may be thought convenient as before; in like Manner the whole may be done by the Removal of the Pole.

When we here ſpeak of Stations we do not mean ſuch as are uſually taken at every particular Angle of the Field: For it is to be apprehended, that every ſkilful Surveyor, particularly ſuch who uſe Calculation, will take the longeſt Diſtances poſſible, not only to leſſen the Number of Stations, for the Eaſe of either Protraction or Calculation, but with greater Certainty to account for the Land paſſed by, on the right Hand or on the left, which is taken by Off-ſets: And ſurely it will be allowed that any Meaſure taken on the Ground and the Content thence arithmetically computed, will be much more accurate than that which is obtained from any Geometrical Projection.

From what has been ſaid it is plain, that from this Method any Fault committed in a Survey can be readily determined, and therefore muſt be much preferable to the preſent Method of taking Dia-

gonals, or the Bearings and Lengths of Lines acroſs Land, to accompliſh that End; which laſt Method is too frequently uſed by Surveyors to approximate or arrive near the Content, which will ever remain uncertain, let theſe Diagonals be ever ſo many, 'till the Station or Stations wherein the Error or Errors were committed, be found; and the Fault or Faults be corrected.

Where one Diagonal is taken it may perhaps cloſe or meet with one Part of the Survey and not with the other; in this Caſe, if the Surveyor would diſcover his Error, he muſt ſurvey that Part of the Land which did not cloſe, and this may be half or more, of the whole. And ſhould the Diagonal cloſe with neither Part, but be too long, or too ſhort, or ſhould it fall on either Side of the aſſigned Point it was to cloſe with, he ought to go over the whole, and make a new Survey of it in order to diſcover his Error.

A Number of Diagonals are frequently taken, the Sum of the Lengths of which very often exceeds the Circuit of the Ground, and after all they are but Approximations, and the Content remains uncertain as before; therefore he who returns a Map, made up by the Aſſiſtance of Diagonals, where there remains a Miſcloſure in any one Part, runs the Riſque of being detected in an Error, and muſt ſuffer Uneaſineſs in his Mind, as he cannot be certain of the Return he makes.

The frequent Miſcloſures which are botched up by Diagonals, occaſion the many and frequent ſcanda-

ſcandalous Broils and Animoſities between Surveyors, which tend to the Loſs of Character of the one or the other, and indeed often to the Diſrepute of both, as well as to that of the Science they profeſs.

But theſe may be eaſily remedied by Interſections, and the Bearing or Line be adjuſted where the Fault was committed, and 'till this be found, nothing can be certain.

To

To enlarge or diminiſh MAPS.

How to enlarge or diminiſh a Map, or how to reduce a Map from one Scale to another: Alſo the Manner of uniting ſeparate Maps of Lands which join each other, into one Map of any aſſigned Size.

LAY the Map you would enlarge, over the Paper on which you would enlarge it, and with a fine protracting Pin, prick thro' every angular Point of your Map, join theſe Points on your Paper, (laying the Map you copy before you) by pencilled or popped Lines, and you have the Copy of the Map you are to enlarge: In this Manner any Protraction may be copied on Paper, Vellum, or Parchment for a fair Map.

If you would enlarge a Map to a Scale which is double, or treble, or quadruple to that of the Map to be enlarged; the Paper you muſt provide for its Enlargement muſt be two, or three, or four Times as long and broad as the Map; for which Purpoſe in large Things you will find it neceſſary to join ſeveral Sheets of Paper, and to cement them with white Wafer or Paſte, but the former is beſt.

Then pitch upon any Point in your copied Map, for a Center; from whence if Diſtances be taken to its extream Points, and thence if thoſe Diſtances be ſet in a right Line with (but from) the Center, and theſe

these last Points fall within your Paper, the Map may be encreased on it to a Scale as large again as its own; and if the like Distances be again set outwards in right Lines from the Center, and if these last Points fall within your Paper, it will contain a Map encreased to a Scale three Times as large as its own, *&c.*

Plate XII. Fig. 2.

Let the pricked or popped Lines represent the Copy of a Down or old Survey, laid down by a Scale of 80 Perches to an Inch, and let it be required to enlarge it to one laid down by 40 to an Inch.

Pitch upon your Center as ⊙, from whence thro' *a* lay the fiducial Edge of a thin Ruler, with a fine pointed pair of Compasses, take the Distance from *a* to the Center ⊙, and lay it by the Ruler's Edge from *a* to A: In the like Manner take the Distance from the next Station *b* to the Center ⊙, and lay it over in a right Line from *b* to B, and join the Points A and B by the right Line AB: In the like Manner set over the Distance from every Station to the Center, from that Station outwards, and you will have every Point to enlarge to; the joining of these constantly as you go on by right Lines, will give you the enlarged Map required.

In taking the Distances from every Station to the Center, set one Foot of the Compasses in the Station, and the other very lightly over the Center-Point, so lightly as scarcely to touch it, otherwise the Center-Point, will become so wide that it may occasion several Errors in the enlarged Map: For if you err from the exact Center but a little, that Error will become double, or treble, or quadruple,

druple, as you enlarge to a Scale that is double or treble or quadruple, of the given one; therefore great Accuracy is required in enlarging a Map.

When you have done with a Station, give a Dash with a Pen or Pencil to it, such as at the Station *a* and *b*; by this Means you cannot be disappointed in missing a Station, or in laying your Ruler over one Station twice.

From what has been said it is plain, that if a Map is to be enlarged to one whose Scale is double the given one, that the Distances from the respective Stations to the Center, being set over by the Ruler's Edge, will give the Points for the enlarged one. And thus may a Map be enlarged from a Scale of 160 to one of 80, from one of 80 to one of 40, from one of 20 to one of 10 Perches to an Inch, *&c.* for to enlarge to a Scale that is double, the Number of Perches to an Inch, for the enlarged Map must be Half of those to an Inch for that to be enlarged: To enlarge to a Scale that is treble the given one, the Number of Perches to an Inch for the enlarged Map, will be one third of those for the other; if to a Scale that is quadruple the given one, the Number of Perches to an Inch, for the enlarged Map, will be one fourth of those for the other, *&c.* therefore if you would enlarge a Map which is laid down by a Scale of 120 Perches to an Inch, to one of 40 Perches to an Inch; the Distance from the several Stations to the Center, being set twice beyond the said Stations, will mark out the several Points required, for these Points will be three Times farther from the Center than the stationary Points of the Map are.

In the same Manner, if you would enlarge a Map from a Scale of 160, to one of 40 Perches to

to the Center, being ſet three Times beyond ſaid Stations, will lay out the Points for your enlarged Map, for theſe Points will be four Times farther from the Center than are the Stations of the Map.

When a Map is enlarged to another, whoſe Scale is double or treble, or quadruple *&c.* of the given one, every Line, as well as the Length and Breadth of the enlarged Map will be double, or treble, or quadruple, *&c.* thoſe of the given one, for it muſt be eaſy to conceive that thoſe Maps are like: But the Area, if the Scale be double, will be four Times; if treble, nine Times; if quadruple, ſixteen Times that of the given Figure; that is, it will contain four, nine, or ſixteen Times as many ſquare Inches as the given one (for it has been ſhewn that like Polygons are in a duplicate Proportion with their homologous Sides.) Yet theſe Figures being caſt up by their reſpective Scales, will produce the ſame Content.

Thus much is ſufficient for enlarging Maps, and from hence, diminiſhing of them will be obvious; for one fourth, one third, or half the Diſtances from the ſeveral Stations to the Center, will mark out Points, which, if joined, will compoſe a Map ſimilar to the given one, whoſe Scale will be four Times, three Times, or twice as ſmall as the given one.

Thus, if we would reduce a Map from 40 to 80, from 20 to 40, from 10 to 20 Perches to an Inch, *&c.* half the Diſtance of the Stations from the Center will give Points requiſite for drawing the Map; if we would reduce from 40 to 120, from 20 to 60, from 10 to 30 Perches to an Inch, *&c.* one third of the Diſtances to the Center, will give the Points for the Map: And if we would reduce

from 40 to 160, from 20 to 80, from 10 to 40 Perches to an Inch, *&c.* one fourth of the Diſtances to the Center, will give the Points for the Map.

By the Methods here laid down I have reduced a Map from a Scale of 40 to one of 20 Perches to an Inch, which contained upwards of 1200 Acres, and conſiſted of 224 ſeparate Diviſions, without the leaſt Confuſion from the Lines; for none can ariſe if the Methods here laid down be ſtrictly obſerved.

I have alſo from the ſame Methods reduced a large Book of Maps, each of which was an entire Skin of Parchment, and the whole contained upwards of 46000 Acres, to a Pocket Volume; and afterwards connected all theſe Maps into one Map, which was contained in one Skin of Parchment: Therefore upon the whole I do recommend theſe Methods for reducing Maps to be much more accurate than any of the Methods commonly uſed, ſuch as ſquaring of Paper, uſing a Parallelogram, proportional Compaſſes, or any other Method I ever met with, though the Figures to be reduced were ever ſo numerous, irregular, or complicated.

How to unite ſeparate Maps of Lands which join each other, into one Map of any aſſigned Size.

If there be ſeveral large Maps contained in a Book, each of which ſuppoſe to take up a Skin of Parchment, or a Sheet of the largeſt Paper; which Maps of Lands join each other; and it be required to reduce them to ſo ſmall a Scale, that all of them when joined together may be contained

in

in one Skin, half a Skin, or any assigned sized Piece of Parchment, or Paper.

Having pricked off and copied the several Maps on any Kind of Paper, unite them by cutting with Scissars along the Edge of one Mearing which is adjoining the other, but not cutting by the Edge of both, and throw aside the Parts cut off; then lay these together on a large Table, or on the Floor, and where the Mearings agree, they will fit in with each other as Indentures do; and after this Manner they are easily connected: Measure then the Length and Breadth of the entire connected Maps, and the Length and Breadth of the Parchment or Paper you are confined to; if the former be three four, or five Times greater, (that is longer and broader) than the latter, reduce each copied Map severally to a Scale that is three, or four, or five Times less, as before; and the same Parts of the Mearings you cut by in the large Maps, by the same you must also cut in small ones, and unite the small as the large ones were united; cementing them together with white Wafer: Thus will your Map be reduced to the assigned Size, which copy over fair, on the Parchment, or Paper you were confined to.

But it is not always that a Person is confined to a given Area of Parchment, or Paper; in such Cases, if there are many large Maps to be united into one, reduce each of them severally to a Scale of 160 Perches to an Inch, and unite those by the Contiguity of Mearings as before: Or if you have a few, it will be sufficient to reduce them to a Scale of 120, *&c.* But having the Maps given, and the Scale by which they are laid down, your Reason will be sufficient to direct you to know, what Scale they should be reduced to.

How to trace LANDS from the DOWN, or any other SURVEY.

IN the Surveyor-General of Land's Office (which is now kept in the ſame Building with the Ordnance-Office, in the lower *Caſtle-Yard*, *Dublin*) are kept what is called the Down Surveys, or Surveys of moſt of the forfeited Lands in this Kingdom. Theſe Surveys were done under the Direction of Sir *William Petty*, Bart. then Surveyor-General of Lands, and afterwards Earl of *Shelburne* about the Year 1641; there was a Surveyor previous to this, called the Civil Survey, or one by Eſtimation; and becauſe the latter of theſe was laid down by Maps on Paper, it thence got the Name of the Down Survey.

About forty Years ago the Treaſury-Office in *Eſſex-Street*, where the Surveyor-General's Office was alſo then kept, was burnt; and though moſt of the Down Surveys were ſaved, yet ſome were conſumed, and this is the Reaſon that the Surveys of all the forfeited Lands are not to be had at the Surveyor-General's Office; but Sir *Thomas Taylor*, Bart. has a fair and true Copy of all the Down Surveys of the forfeited Lands, which often proves of great Uſe to Gentlemen; though the Down Surveys only,

only, are thoſe allowed by the Laws of the Land to be final and deciſive.

It will not here be unneceſſary to obſerve, that 6*s*. 8*d*. is to be paid for the Copy of every Denomination, but nothing for a Search, if it be found; otherwiſe 5*s*. is to be paid for the Search, if the Denomination cannot be found, that is, if it were not forfeited or burnt.

The Differences generally ariſing about Lands wherein Down Surveys are found neceſſary, are for the moſt Part of this, or the like Nature.

A, who reſides in the Country upon his own Eſtate, takes Lands contiguous thereto from B, who reſides in the City or far from his Eſtate, and who perhaps never ſaw it. A in ſome Time defaces the contiguous Mearings, and runs others within ſide of B's Ground, ſometimes nearly ſimilar, and parallel to the defaced Mearing. At the Expiration of the Leaſe, A, or his Heir (who may be innocent of the Matter,) delivers to B, or his Heir, the Lands according to the new run Mearings; but the latter by a Survey he cauſes to be made, in order to ſet it to a new Tenant, finds his Land widely defective of the Number of Acres demiſed to A, which alarms him; and upon applying to A, without finding any Redreſs, a Law-Suit is commenced, and the Court orders a Trace to be run from the Lines of the Down Survey: If it is to be had, this readily diſcovers the Fraud, if not, the Matter is determined by the Evidences of the oldeſt Perſons of honeſt Repute in the Neighbourhood.

The beſt Way to avoid all future Diſputes concerning Lands, is to have the Lands Surveyed before

fore ſet, and to annex Maps thereof to the Leaſes. If afterwards the Mearings ſhould be defaced or changed, or any Diſpute ſhould ariſe concerning them; the Map to the Leaſe may be eaſily traced, and the Difference eaſily reconciled.

A Down Survey is thus Traced.

Take a Survey of the Land, as it is ſhewn you by the beſt Information you can receive, and make a Protraction thereof upon parallel Paper, by a Scale of 40 Perches to an Inch.

Then enlarge your Down Survey to a Scale of 40 as before, (provided it be not already laid down by that Scale, but they are oftner laid down by a Scale of 80, of 120, and ſometimes of 160 Perches to an Inch) for it is not ſafe to enlarge the Down Survey from any ſmall Scale to one larger than 40, becauſe the Errors of the Diſtances from the ſtationary Points if ever ſo minute, will be thereby much encreaſed.

I cannot omit here to obſerve, that ſome in order to enlarge the Down Survey, have pricked it off on Parallel Paper, and after drawing the Map thereon, have (by producing every Diſtance Line) found their Bearings, and by meaſuring the Lines from the given Scale, have formed a Field-Book: And after all this Trouble, when they have protracted it to the aſſigned Scale, all was wrong, for it would not meet or cloſe: Of this I have ſeen many Inſtances; yet the Surveyor would make it do, as Miſcloſures are many Times forced to do; the Conſequence of either, the moſt unſkilful Surveyor cannot be ignorant of.

Let

Let the Enlargement you make of the Down Survey to the like Scale of your Protraction, be upon the thinneſt Paper you can get, which rub over with any Kind of Oil, to make it tranſparent, but wipe off the Oil that it may not ſmeer the Protraction. If Oil of Turpentine can be readily had, it will do beſt, becauſe it won't ſmeer; but in Caſes of Extremity, Butter turned to Oil will do

Then apply your oiled Map, or Enlargement of the Down Survey, upon your protracted Map, and you will readily ſee what Points of the one will coincide with thoſe of the other; and when you have brought the greateſt Number of Points in each to agree that is poſſible, the Maps are then applied to the greateſt Advantage: See then where the Lines of the Down Survey run within or without thoſe of your Protraction, and prick them on it; and to diſtinguiſh them from the black Lines of the Protraction, let them be drawn in red Ink, or popped.

Produce every ſtationary Line of the Down Survey, (or every red, or popped Line) forwards, that runs within or without Side of the Protraction; then if the Center of your Protractor be applied to every ſtationary Point of ſuch continued Lines, and your Protractor be kept Parallel to the Parallels of Meridians on the Protraction; the Continuation of every ſtationary Line, will point out on the Protractor's Edge, the Degree of Bearing of ſuch Lines: If theſe Bearings be inſerted in a Field-Book, as well as the Lengths of the ſeveral ſtationary Lines, (which are eaſily obtained by meaſuring them from the Scale by which they were laid down,) that Field-Book will direct your Trace.

Being

Being thus prepared, go to the Ground and ſet up your Inſtrument at the firſt Station of ſaid Field-Book, and turn the Inſtrument about 'till you make the Needle (if you uſe a Circumferentor) point to the firſt Bearing in your Field-Book; (or if you uſe a Theodolite ſet it North and South by the Needle, and bring the Index to cut the ſaid Degree or Bearing): Let the Inſtrument ſtand in this Direction, 'till you ſend forth ſome Perſon to a Diſtance, greater if poſſible than the aſſigned one, or that of your ſtationary Line; then look through the Sights which govern the Direction, and waſt your Hat to one or to the other Side, as a Token to the Perſon you ſent forward to ſtand more to the right or to the left, and continue to give the Signal 'till you have him in the Direction of the Sights, and then put on your Hat as a Signal to him that he is in the true Direction; there let him continue 'till you have meaſured the ſtationary Line in your Field-Book towards him, and at the End thereof cauſe a large Hole to be dug, or a Stake to be driven, and your firſt Line is traced.

In like Manner proceed to trace every Diſtance Line, cauſing a large Hole to be made, or a Stake to be driven, or both, at every Station; 'till you have laid out and traced all your Field-Notes: And when Lines, or Ditches, are run from one Hole, or Stake to another, they will be the true Mearings, or Boundaries of the Land.

The Area comprehended between the popped, or red Lines, and the black ones of your Protraction, will be what is gained or loſt.

This is the beſt Method of tracing Down Surveys; and it is plain, that the ſame or a like Application may be made of any Survey whatſoever, ſo as to run a Trace thereof.

The

The Variation of the Compass;

And how to find it by Amplitudes or Azimuths of the Sun.

1. IT was before obſerved, that the Needle does not point truly to the North or South Points of the Horizon: The Number of Degrees therefore, that the Points of the Needle, are from the North or South Points of the Horizon, is called the *Variation of the Needle*, or *Compaſs*.

This Variation differs widely in many Places; for in ſome, the Needle will point ſeveral Degrees on the Weſt Side of the North; at others there will be little or no Variation, and again, at others it will point ſeveral Degrees on the Eaſt Side; in the ſame Place it differs ſenſibly in a few Years: The true Cauſe or Theory of which, has not hitherto been diſcovered or explained for Want of a ſufficient Number of Obſervations.

2. The Globe of the Earth revolves round its Axis in twenty-four Hours from Weſt to Eaſt, and hence all Celeſtial Bodies ſeem to move from Eaſt to Weſt.

3. The Extremities of the Axis are called the *Poles*; the one the North or *Arctic*, and the other the South or *Antarctic*. And if the Axis be produced to the Heavens, it will point out the *Celeſtial Poles*.

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4. If a Circle be ſuppoſed to paſs round the Globe of the Earth, ſo as to be equidiſtant from each Pole, it is called the *Equator*, or *Equinoctial Line*, and by ſome the *Line* only.

And if the Plane of the Equator be produced to the Heavens, it will lay out the *Celeſtial Equator*.

5. *The Latitude of any Place*, is its neareſt Diſtance to, and counted from the Equator in Degrees and Minutes; and is North or South as it lies on the North or South Side of the Equator.

6. The Poles are 90 Degrees from the Equator, therefore the *Complement of the Latitude of any Place*, is the Latitude taken from 90 Degrees, or the Diſtance of the Place from its neareſt Pole.

7. *The Declination of the Sun*, is the neareſt Diſtance thereof from the celeſtial Equator counted in Degrees and Minutes; and is North or South, as it lies on the North or South Side of the Equator.

8. The Sun's Declination taken from 90, leaves the *Complement* thereof; or its Diſtance from the neareſt celeſtial Pole.

9. *The Sun's Altitude*, is the Number of Degrees and Minutes the Sun is above the Horizon, and is eaſily found by a Quadrant as before.

10. What the Sun's Altitude wants of 90, or the Sun's Diſtance from the *Zenith* or Point of the Heavens perpendicularly over you, is *the Complement of the Altitude*.

11. *The Magnetical Amplitude*, is the Complement of the Sun's Bearing at Rising or Setting, taken by the Quarter'd-Compass; or it is the Number of Degrees the Sun is from the East or West Point of the Compass, at Rising or Setting.

12. *The True Amplitude* is the Complement of Degrees the Sun would rise or set on if the Compass did not vary; or it is the Number of Degrees the Sun is from the East or West Point of the Horizon, at Rising or Setting; and this true Amplitude is always North, if the Sun's Declination be North, or South if the Sun's Declination be South.

To find the Variation by the Amplitudes.

Having the Latitude of the Place, and the Sun's Declination given, the true Amplitude is found by this Astronomical Proportion, *viz.*

As the Co-sine or Sine Complement of the Latitude;
Is to the Sine of the Sun's Declination,
So is Radius
To the Sine of the True Amplitude.

Then if both Amplitudes be North, or both South, their Difference is the Variation, but if one be North and the other South, their Sum is the Variation.

To know whether the Variation be Easterly or Westerly.

Let the Observer turn his Face to the Sun, then if the true Amplitude be to the right Hand of the

magnetical one, the Variation is Easterly, but if to the left, Westerly.

Example I.

On the 17th Day of *May* 1752, the Sun's Bearing at Rising, being NE 71°, in the Latitude $53^{\circ}.\,20^{\prime}$ N. required the true Amplitude, and the Variation of the Needle.

Because the annexed Table is for the *New-Stile*, which is 11 Days before the *Old*; we find the Sun's Declination for the 28th of *May* in the Leap Year (for this Table does not take Place before 1753) to be $21^{\circ}.34^{\prime}$ N.

After the same Manner this Table may serve 'till 1753.

To find the True Amplitude.

As the Co-sine of the Latitude	$36^{\circ}.40^{\prime}$	9.77609
Is to the Sine of the Declination	$21^{\circ}.34$ N	9.56536
Radius	90.00	10.00000
To the Sine of the true Ampl.	38.00 N	9.78927

90°---71°=19° the Mag. Amp. from the East.

True Amplitude E $38^{\circ}.00^{\prime}$ N for the Decl. is N.
Magnetical Ampl. E 19.00 N

Variation 19.00 W because the true Amplitude is to the left of the magnetical.

EXAM-

Example II.

Suppose the Sun's true Amplitude is found to be W $42^{\circ}.00'$ S. and the magnetical Amplitude W 23.00 S. the Sun's Bearing at setting being SW. 67°. Required the Variation.

90°---67°=23° the magnetical Amplitude from the West.

True Amplitude	W $42^{\circ}.00'$	S
Magnetical Amplitude	W 23 .00	S
Variation	19.00	W

In this Case also the true Amplitude is to the left of the magnetical; and therefore the Variation is westerly.

Example III.

Sun's Bearing at Rising being SE $77^{\circ}\frac{1}{2}$, and the true Amplitude being found to be E 10. $12'$ N. required the Variation.

90°---$77^{\circ}\frac{1}{2}$ = $12^{\circ}\frac{1}{2}$ the magnetical Amplitude from the East.

True Amplitude	E $10^{\circ}.12'$	N
Magnetical Amplitude	S 12 .30	S
Variation	22.42	W

The true Amplitude being still to the left, the Variation is westerly.

Exam-

EXAMPLE IV.

Sun's Bearing at Setting is SW 81°½, and the true Amplitude is found to be W 6°. 16'. N. Required the Variation.

90°---81°½=8°½ the magnetical Amplitude from the West.

True Amplitude	W	6°.16' N
Magnetical Amplitude	W	8 .30 S
	Variation	14.46 E

The true Amplitude being to the right of the magnetical, the Variation is easterly.

2. *To find the Variation by Azimuths.*

13. The Sun's magnetical Azimuth is the Bearing thereof at any Time of the Day, taken by the Quarter'd-Compass; that is counted from the North or South towards the East or West Points of the Box.

14. The Sun's true Azimuth is the Point of the Compass it would bear from you upon, if there was no Variation; or it is the Distance intercepted between the North or South Points of the Horizon, and a vertical Circle, or Circle drawn from the Zenith through the Sun to the Horizon.

Having the Latitude of the Place, the Sun's Declination, and its Altitude given, the true Azimuth is obtained by the following astronomical Proportions.

1. *As the Tangent of half the Complement of the Latitude*

Is to the Tangent of half the Sum of the Distance of the Sun from the Pole, and Complement of the Altitude,

So is the Tangent of half the Difference between the Distance of the Sun from the Pole, and Complement of the Altitude,

To the Tangent of a fourth Arc.

Add this fourth Arc and Half the Complement of the Latitude together, their Sum will give a fifth Arc; from which if the Complement of the Latitude be taken, the Remainder will give a ſixth Arc. Then ſay,

As Radius
Is to the Tangent of the Altitude,
So is the Tangent of the ſixth Arc
To the Co-ſine of the Sun's true Azimuth.

Which is counted from the North or South, to the Eaſt or Weſt, according to the Sun's Situation at the Time and Place of Obſervation.

If the Latitude of the Place and the Sun's Declination are both North or both South, the Declination taken from 90°, gives the Sun's Diſtance from the Pole; but if one be North and the other South, the Declination added to 90° will give the Sun's Diſtance from that Pole which is neareſt the Obſerver.

If both Azimuths are Eaſt or Weſt, their Difference is the Variation, but if one be Eaſt, and the other Weſt, their Sum is the Variation.

To

To know whether the Variation be Easterly or Westerly.

Just as with the Amplitudes, let the Observer's Face be turned to the Sun; then if the true Azimuth be to the right Hand of the magnetical one, the Variation is Easterly, but if to the left, Westerly.

Example I.

In the Latitude 53° 20' N, the Sun's Declination being 19°.03' N, I find by Observation the Sun's Altitude to be 37°.30' and its Magnetical Azimuth to be SE 51°. Required the Variation.

90°---53°.20'=36°.40' the Compt. of the Latitude
18.20, ½ the Compt. of the Latitude
90°---37°.30'=52°.30'. the Compt. of the Altitude.

90°---19°.03'=70°.57' the Sun's Dis. from the Pole
52.30 Compt. of the Altitude

123.27 Sum

61.43 Half Sum

18.27 Difference

9.13 Half Difference

As the Tang. of ½ the Comp. of the Latitude,	18°.20′--- 9.52031
Is to the Tangent of ½ the Sum of the Distance of the Sun from the Pole and Complement of the Altitude,	61. 43---10.26916
: : Tang. of ½ their Difference	9. 13--- 9.21022
	19.47938
To Tangent of a 4th Arc.	42. 18--- 9 95907

Half the Comp. of the Latitude	18°.20′
The 4th Arc	42. 18
Their Sum is the 5th Arc	60. 38
Complement of the Lat. subtract	36. 40
Gives the 6th Arc	23. 58

As Radius	90°.00′---10.00000
Is to the Tang. of the Alt.	37. 30--- 9 88498
: : Tangent of the 6th Arc	23 58--- 0.64790
Co-sine of the Sun's true Azim.	70. 04--- 9.53288

True Azimuth	S 70°.04′ E
Magnetical Azimuth	S 51.00 E
Variation	19. 04 W

The true Azimuth being to the left of the magnetical one, the Variation is westerly.

Example II.

Suppose the Sun's true Azimuth N 82°.20' E, but the magnetical one N 70°.30' E. Required the Variation.

True Azimuth	N	83°.20'	E
Magnetical Azimuth	N	70.30	E
Variation		12.50	E

The true Azimuth being to the right of the magnetical one, the Variation is easterly.

Example III.

Suppose the Sun's true Azimuth were S 37°. 15' W. and the magnetical one S 44°.20' W. Required the Variation.

True Amplitude	S	37°.15'	W
Magnetical Azimuth	S	44. 20	W
Variation		6.05	W

The true Azimuth being to the left of the magnetical one, the Variation is westerly.

Example IV.

Suppose the Sun's true Azimuth be S 4°.05' W, and the magnetical one S 3°. 30' E. Required the Variation.

True

True Azimuth	S	4°.95'	W
Magnetical Azimuth	S	3.30	E
Variation		7.35	E

The true Azimuth being to the right of the Magnetical, the Variation is Eaſterly.

The Variation of the Compaſs was firſt obſerved at *London*, in the Year 1580, to be one Point of the Compaſs Eaſterly, or 11°.15' E, after which Time it became leſs; for in the Year 1622 it was 6°.00 E, in 1634 it was 4°.05 E, and ſo continued to decreaſe 'till the Needle coincided with the true Meridian, and then there was no Variation; after which the Variation became Weſterly, and has ever ſince increaſed to the Weſtward: for in the Year 1672 it was 2°.30' W; in the Year 1683 it was 4°.30' W at *London*; in 1722 it was at *Dublin* found to be 11°.15' W, and in 1751 it was there found to be 19°.00 W; but how far it will continue to move more Weſterly, Time and Obſervations will probably be the only Means to diſcover.

At *Paris* in 1640, the Variation was 3.00' E; in 1666 there was no Variation; but in 1681 it was 2°,30 W, and ſtill continues to go on Weſterly.

How to draw a true Meridian Line to a Map, having the Variation and Magnetical Meridian given.

On any Magnetical Meridian or Parallel, upon which your Map is protracted, ſet off an Angle from the North towards the Eaſt, equal to the Degrees or Quantity of Variation, if it be Weſterly, or from

the North towards the West if it be easterly, and the Line which constitutes such an Angle with the Magnetical Meridian, will be a true Meridian Line.

For if the Variation be westerly, the Magnetical Meridian will be the Quantity of Variation of the West Side of the true Meridian, but if Easterly on the East Side, therefore the true Meridian must be a like Quantity on the East Side of the Magnetical one, when the Variation is westerly, and on the West Side when it is easterly.

How to lay out a true Meridian Line by the Circumferentor.

If the Variation be westerly, turn the Box about 'till the North of the Needle points as many Degrees from the *Flower-de-Luce* towards the East of the Box, or 'till the South of the Needle points the like Number of Degrees from the South towards the West, as are the Number of Degrees contained in the Variation, and the Index will be then due North and South; therefore if a Line be struk out in the Direction thereof, it will be a true Meridian Line.

If the Variation was easterly, let the North of the Needle point as many Degrees from the *Flower-de-Luce* towards the West of the Box, or let the South of the Needle point as many Degrees towards the East, as are the Number of Degrees contained in the Variation, and then the North and South of the Box, will coincide with the North and South Points of the Horizon, and consequently a Line being laid out by the Direction of the Index will be a true Meridian Line.

This

This will be found to be very useful in setting an Horizontal Dial, for if you lay the Edge of the Index by the Base of the Stile of the Dial, and keep the angular Point of the Stile towards the South of the Box, and allow the Variation as before, the Dial will then be due North and South, and in its proper Situation; provided the Plane upon which it is fixed be duly horizontal, and the Sun be South at Noon; but in Places where it is North at Noon, the Angular Point of the Index must be turned to the North.

How Maps may be traced by the Help of a true Meridian Line.

If all Maps had a true Meridian Line laid out upon them, it would be easy by producing it, and drawing Parallels, to make out Field Notes; and by knowing the Variation, and allowing it upon every Bearing and having the Distances, you would have Notes sufficient for a Trace. But a true Meridian Line is seldom to be met with, therefore we are obliged to have Recourse to the foregoing Method. It is therefore adviseable to lay out a true Meridian Line upon every Map.

A TABLE *of the Sun's Declination.*

Days	Jan. S.	Feb. S.	March. S.	April N.	May N.	June N.	Days
	° '	° '	° '	° '	° '	° '	
For the Years 1753, 1757, 1761, 1765, 1769, 1773.							
4	22.43	16.08	S 6.19	5.48	16.03	22.29	4
8	22.13	14.54	4.46	7.18	17.10	22.54	8
12	21.37	13.36	3.12	8.47	18.12	23.12	12
16	20.54	12.14	1.37	10.13	19.10	23.23	16
20	20.04	10.49	0.03	11.36	20.03	23.29	20
24	19.09	9.21	N 1.32	12.57	20.49	23.27	24
28	18.08	7.51	3.06	14.14	21.31	23.19	28
For the Years 1754, 1758, 1762, 1766, 1770, 1774							
4	22.45	16.13	S 6.25	5.42	15.58	22.27	4
8	22.16	14.58	4.52	7.13	17.05	22.53	8
12	21.39	13.40	3.18	8.41	18.08	23.11	12
16	20.57	12.19	1.43	10.08	19.06	23.23	16
20	20.07	10.54	0.09	11.31	19.59	23.29	20
24	19.12	9.26	N 1.26	12.52	20.47	23.27	24
28	18.11	7.54	3.00	14.09	21.29	23.20	28
For the Year 1755, 1759, 1763, 1767, 1771, 1775.							
4	22.46	16.17	S 6.31	5.34	15.54	22.26	4
8	22.17	15.03	4.58	7.07	17.02	22.51	8
12	21.42	13.45	3.24	8.36	18.05	23.10	12
16	20.59	12.24	1.49	10.03	19.03	23.23	16
20	20.10	10.59	0.14	11.27	19.56	23.28	20
24	19.16	9.32	N 1.20	12.47	20.44	23.28	24
28	18.15	8.02	2.54	14.05	21.66	23.21	28
For the Years 1756, 1760, 1764, 1768, 1772, 1776							
4	22.48	16.21	S 0.13	5.54	16.07	22.31	4
8	22.20	15.08	4.40	7.24	17.14	22.55	8
12	21.44	13.50	3.06	8.53	18.16	23.13	12
16	21.02	12.29	1.31	10.19	19.14	23.24	16
20	20.14	11.04	N 0.03	11.42	20.06	23.29	20
24	19.19	9.37	1.38	13.02	20.52	23.27	24
28	18.19	8.08	3.12	14.19	21.34	23.18	28

A TABLE *of the Sun's Declination.*

Days	July N.	August N.	Sept. N*	Oct. S.	Nov. S.	Dec. S.	Days
	° ′	° ′	° ′	° ′	° ′	° ′	
Each being the first Year after Leap-Year.							
4	22.55	17.15	N 7.09	4.26	15.29	22.20	4
8	22.30	16.09	5.39	5.58	16.41	22.48	8
12	22.00	14.58	4.08	7.30	17.48	23.09	12
16	21.23	13.44	2.35	8.59	18.51	23.23	16
20	20.41	12.26	1.02	10.27	19.48	23.29	20
24	19.53	11.05	S 0.32	11.52	20.39	23.27	24
28	19.00	9.41	2.06	13.14	21.24	23.18	28
Each being the second Year after Leap-Year.							
4	22.56	17.19	N 7.14	4.20	15.25	22.17	4
8	22.32	16.13	5.44	5.53	16.37	22.46	8
12	22.02	15.02	4.13	7.24	17.44	23.08	12
16	21.26	13.48	2.41	8.54	18.47	23.22	16
20	20.44	12.31	1.07	10.21	19.45	23.28	20
24	19.57	11.10	S 0.26	11.47	20.36	23.27	24
28	19.04	9.46	2.00	13.09	21.22	23.19	28
Each being the third Year after Leap-Year.							
4	22.57	17.23	N 7.20	4.15	15.20	22.16	4
8	22.34	16.17	5.50	5.47	16.33	22.45	8
12	22.04	15.07	4.19	7.19	17.41	23.07	12
16	21.28	13.53	2.46	8.58	18.43	23.21	16
20	20.46	12.36	1.13	10.16	19.41	23.28	20
24	20.01	11.15	S 0.20	11.42	20.33	23.28	24
28	19.07	9.51	1.54	13.04	21.19	23.20	28
Each being Leap-Year.							
4	22.54	17.18	N 7.03	4.32	15.34	22.22	4
8	22.29	16.04	5.33	6.05	16.46	22.49	8
12	21.58	14.53	4.01	7.36	17.53	23.10	12
16	21.21	13.39	2.29	9.05	18.55	23.23	16
20	20.38	12.21	0.55	10.33	19.52	23.29	20
24	19.50	10.59	S 0.38	11.58	20.42	23.27	24
28	18.56	9.35	2.12	13.19	21.27	23.17	28

To reduce one MEASURE to another.

PROB. I.

To reduce Irish to English Acres, and the contrary.

IT is plain, that if any Parcel of Ground be measured by Chains of different Lengths, that the Figures constructed from these different Measures will be unequal, but like to each other; and therefore such Figures will be in a Duplicate Proportion to their homologous Sides, by Theo. 23. Sect. 1.

Let (in Plate 2. Fig. 6) *abcde* be a Map of Ground that was measured by an *Irish* or Plantation Chain, and ABCDE be another of the same Ground that was measured by an *English* or Statute Chain; if the Line *ab* in the first be 11 *Irish* Perches, the Line AB in the other will be 14 *English* ones; for it has been already shewn in Page 135, that 11 *Irish* Perches are equal in Length to 14 *English* ones. Whence it is plain that.

1. *If you would reduce Irish Acres to English ones, it will be*

As the Square of 11, *or* 121,
Is to the Area in Irish Acres,
: : *The Square of* 14 *or* 196,
To the Area in English Acres.

2. *And if you would reduce English Acres to Irish ones, it will be*

As the Square of 14, *or* 196,
Is to the Area in English Acres,
: : *The Square of* 11, *or* 121,
To the Area in Irish Acres.

EXAMPLE I.

A. R. P.
Reduce 246. 3. 14 Plantation Measure to Statute Measure

4
987
40
39494

```
          P.
121 : 39494 : : 196
          196
      ------
      236964
     355446
     39494      160
     ------              A.   R.   P.
121)7740824(63973(399.   3.   13
    ---      ----
    480      1597
    ---      ----
    1178     1573
    ----     ----
     892   40)133(3R
     ---      ---
     454      13(P
     ---      ---
      91
      --
```

Answer 339A. 3R. 13P. Statute Measure.

EXAMPLE

Example II.

```
         A.   R.  P.
Reduce 416. 2. 14 Stat. Measure to Plantation
Measure    4
        ----
        1666
          40
        ----
       66654
       -----

           P.
196 : 66654 : : : 121
            121
          -----
          66654
        133308
        66654     160
        ------             A.   R.  P.
196)8065134(41148(257. 0. 28.
    ---      ----
    225      914
    ---      ---
    291     1148
    ---     ----
    953       28(P
   ----       --
   1694
   ----
    126
    ---
```

Answer 257A: oR. 28P. Plantation Measure.

PROB. II.

How to reduce Plantation Measure to Cuningham Measure, and the contrary.

Since 7 Yards or 28 Quarters make an Irish, and $6\frac{1}{4}$ Yards or 25 Quarters a Cuningham Perch; therefore 25 Irish Perches, are equal Length to 28 Cuningham ones: For $25 \times 28 =$ the Quarters in 25 Irish Perches, will be $= 28 \times 25$, the Quarters in 28 Cuningham Perches. Then as before

1. *If you would reduce Irish Acres to Cuningham ones it will be*

As the Square of 25, *or* 625
Is to the Area in Irish Acres,
: : *The Square of* 28, *or* 784,
To the Area in Cuningham Acres.

2. *If you would reduce Cuningham Acres to Irish ones, it will be*
As the Square of 28, *or* 784,
Is to the Area in Cuningham Acres,
: : *The Square of* 25, *or* 625,
To the Area in Irish Acres.

EXAMPLE I.

A. R. P.
In 321. 3. 20 Irish, how many Cuningham Acres?

A. R. P.
321. 3. 20.=51500

625

P. P.
625 : 51500 : : 784 : 64602

P. A. R. P.
64602=403. 3. 02 Anſwer.

EXAMPLE II.

A. R. P.
In 403. 3. 02. of Cuningham Meaſure, how many Iriſh Acres?

A. R. P. P.
403. 3. 02=64602

P. P.
784 : 64602 : : 625 : 51500

P. A. R. P.
51500 = 321. 3. 20 Anſwer.

PROB. III.

How to reduce Statute Meaſure into Woodland Meaſure, and the contrary.

Since $5\frac{1}{2}$ Yards or 11 half Yards make a Statute Perch, and 6 Yards or 12 half Yards, one of Woodland Meaſure; therefore 12 Perches of Statute Meaſure will be equal in Length to 11 of Woodland Meaſure. Then

1. *If you would reduce Statute Acres to thoſe of Woodland, it will be,*

As the Square of 12, *or* 144,
Is to the Area in Statute Acres,
: : *The Square of* 11, *or* 121,
To the Area in Woodland Acres.

2. *If*

2. *If you would reduce Woodland Acres to Statute ones, it will be,*

As the Square of 11, *or* 121,
Is to the Area in Woodland Acres,
: : The Square of 12, *or* 144,
To the Area in Statute Acres.

Example I.

In 206. 2 12. (A. R. P.) Statute Measure, how much of Woodland Measure?

A.	R.	P.		P.
206.	2.	12	=	33052

	P.			P.
144 :	33052	: :	121 :	27773

P.		A.	R.	P.	
27773	=	173.	2.	13	Answer.

Example II.

In 173. 2. 13 (A. R. P.) of Woodland Measure, how many Statute Acres?

A.	R.	P.		P.
173.	2.	13	=	27773

	P.			P.
121 :	27773	: :	144 :	33052

P.		A.	R.	P.	
33052	=	206.	2.	12	Answer.

PROB.

PROB. IV.

How to reduce Statute Measure to Churchland Measure, and the contrary.

This is done by the same Proportions used for Statute and Plantation Measure, the Length of the Perch of Churchland Measure being the same with the Plantation Perch.

PROB. V.

How to reduce Statute to Forest Measure, and the contrary.

Because $5\frac{1}{2}$ Yards or 11 half Yards make a Statute Perch, and 8 Yards or 16 half Yards a Perch of Forest Measure; therefore 16 Perches of Statute Measure, will be equal in Length to 11 Perches of Forest Measure; and then,

1. *If you would reduce Statute Acres, to those of Forest Measure, it will be*

As the Square of 16, *or* 256,
Is to the Area in Statute Acres,
: : *The Square of* 11, *or* 121,
To the Area in Forest Acres.

2. *If you would reduce Forest Acres to Statute ones, it will be*

As the Square of 11, *or* 121,
Is to the Area in Forest Acres,
: : *The Square of* 16, *or* 256,
To the Area in Statute Acres.

In

In 200 Statute Acres, how many Forest ones?

A. A. R. P.
256 : 200 : : 121 : 94. 2. 5 Answer.

A. R. P.
In 94. 2. 05. of Forest Measure, how many Statute Acres?

A. R. P. P.
94. 2. 05=15125

P. P.
121 : 15125 : : 256 : 32000

P.
32000=200 Acres. Answer.

PROB. VI.

How to reduce Statute to Scotch Measure, and the contrary.

The English Perch being $5\frac{1}{2}$ Yards, or $5\frac{3}{6}$, that is 33 sixths of a Yard; and the Scotch one being $6\frac{1}{6}$ Yards, or 37 Sixths of a Yard; therefore 37 Statute Perches are equal in Length to 33 Scotch Perches. And then,

1. *If you would reduce Statute to Scots Acres, it will be,*

As the Square of 37, *or* 1369,
Is to the Area in English Acres,
: : *The Square of* 33, *or* 1089.
To the Area in Scots Acres.

2. *If*

2. *If you would reduce Scots to Statute Acres it will be*

As the Square of 33, *or* 1089,
Is to the Area in Scots Acres,
: : *The Square of* 37, *or* 1369,
To the Area in English Acres.

EXAMPLE I.

A. R. P.
In 96. 3. 12. Statute Meaſure, how many Scots Acres?

A. R. P. P.
96 . . 3 : 12 = 15492

P. P.
1369 : 15492 : : 1089 : 12323

P. A. R. P.
12323 = 77 : 0 : 03 Anſwer.

A. R. P.
In 77 : : 0 : : 03 Scotch Meaſure, how many Statute Acres?

A. R. P. P.
77 : : 0 : : 03 = 12323

P. P.
1089 : : 12323 : : 1369 : 15492

P. A. R. P.
15492 = 96 : : 3 : : 12 Anſwer

Three very uſeful PROBLEMS.

PROB. I.

A Map with its Area being given, and its Scale omitted, to be either drawn or mentioned; to find the Scale.

CAST up the Map by any Scale whatſoever, and it will be,

As the Area found
Is to the Square of the Scale by which you caſt up
: : The given Area of the Map
To the Square of the Scale by which it was laid down.

The Square Root of which will give the Scale.

EXAMPLE.

A Map whoſe Area is 126A. 3R. 16P. being given; and its Scale omitted to be either drawn or mentioned; to find the Scale.

Suppoſe this Map was caſt up by a Scale of 20 Perches to an Inch, and the Content thereby produced be 31A. 2R. 34P.

As

As the Area found 31A. 2R. 34P.=5074P.
Is to the Square of the Scale by which it was cast up, that is to 20 x 20= } 400
: : *The given Area of the Map* 126A. 3R. 16P. =20296P.
To the Square of the Scale by which it was laid down.

5074 : 400 : : 20296 : 1600 the Square of the required Scale.

Root
1600(40
16

)00

Anſwer. The Map was laid down by a Scale of 40 Perches to an Inch.

PROB. II.

How to find the true Content of a Survey, though it be taken by a Chain that is too long or too ſhort.

Let the Map be conſtructed, and its Area found as if the Chain was of a true Length. And it will be

As the Square of the true Chain
Is to the Content of the Map
: : *The Square of the Chain you ſurveyed by*
To the true Content of the Map.

EXAMPLE.

If a Survey be taken with a Chain which is 3 Inches too long; or with one whose Length is 42 Feet 3 Inches, and the Map thereof be found to contain 920 A. 2R. 20P. Required the true Content.

As the Square of 42 F. 0 In.=*the Square of* 504 *Inches* = 254016
Is to the Content of the Map 920 A. 1R. 20P. = 147260P.
: : *The Square of* 42F. 3 In=*the Square of* 507 *Inches*=257049
To the true Content.

	P.		P.
254016 :	147260 : :	257049 :	149019

```
                A.    R.   P.
160)149019(931 . 1 . 19  Answer.
    ------
     501
     ---
     219
     ---
  40)59(1R.
     ---
     19 P.
```

PROB.

PROB. III.

How to divide Land, or to take off any given Part from a Map.

Plate XII. Fig. 1.

Let ABCD, *&c.* be a Map of Ground, containing 11 Acres, it is required to cut off a Piece as DEFGID, that ſhall contain 5 Acres.

Join any two oppoſite Stations as D and G, with the Line DG, (which you may nearly judge to be the Partition Line) and find the Area of the Part DEFG, which ſuppoſe may want 3R. 20P. of the Quantity you would cut off: Meaſure the Line DG, which ſuppoſe to be 70 Perches. Divide 3R. 20P. or 140P. by 35, the $\frac{1}{2}$ of DG, and the Quotient 4 will be a Perpendicular for a Triangle whoſe Baſe is 70, and the Area 140P. Let HI be drawn parallel to DG, at the Diſtance of the Perpendicular 4, and from 3 where it cuts the Mearing, draw a Line to D, and that Line DI will be the diviſion Line; or a Line from D to H will have the ſame Effect; all which muſt be evident from what has been already ſaid.

Some neceſſary Directions concerning Surveys in general.

If you have a large Quantity of Ground to ſurvey, which conſiſts of many Fields or Holdings, and that it be required to map and give the reſpective Contents of the ſame. It is beſt to make a Survey of the Whole firſt, and to be ſatisfied that it is truly taken.

taken, as well as to find its Content; and as you go round the Land to make a Note on the Side of your Field-Book at every Station where the Mearing of any particular Field or Holding intersects or meets the Surround; then proceed from any one of those Stations, and in your Field-Book say, *proceed from such a Station*, and when you have gone round that Field or Division, insert the Station you close at, and so through the whole; a little Practice can only render this sufficiently familiar, and the Method of Protraction must be evident from the Field Notes. When the whole is protracted, and you are satisfied of the Closes of the particular Divisions, cast up each severally, and if the Sum of their Contents be equal to the Content of the whole first found, you may safely conclude that all is right.

The Protraction being thus finished and cast up, transfer it on clean Paper, Vellum or Parchment, as before; be careful to draw your Lines with a fine Pen, write on it the Names of the circumjacent Lands, and set No. 1, 2, 3, 4, &c. in every particular Field or Division; let every Tenant's particular Holding be distinguished by a different coloured Paint being run finely along the Boundaries; let all the Roads, Rivulets, Rivers, Bridges, Bogs, Loughs, Houses, Castles, Churches, Beacons, (or whatever else may be remarkable on the Ground) be distinguished on the Map. Write the Title of the Map in a neat Compartment either drawn, or done from a good Copper-Plate graving, with the Nobleman's or Gentleman's Arms. Prick off one of your Parallels with the Map, and on it make a Mariner's Compass, and draw a *Flower de Luce* to the North, and this will represent the magnetical North, after which set off the Variation, which express in Figures, and through the Center of the Compass,

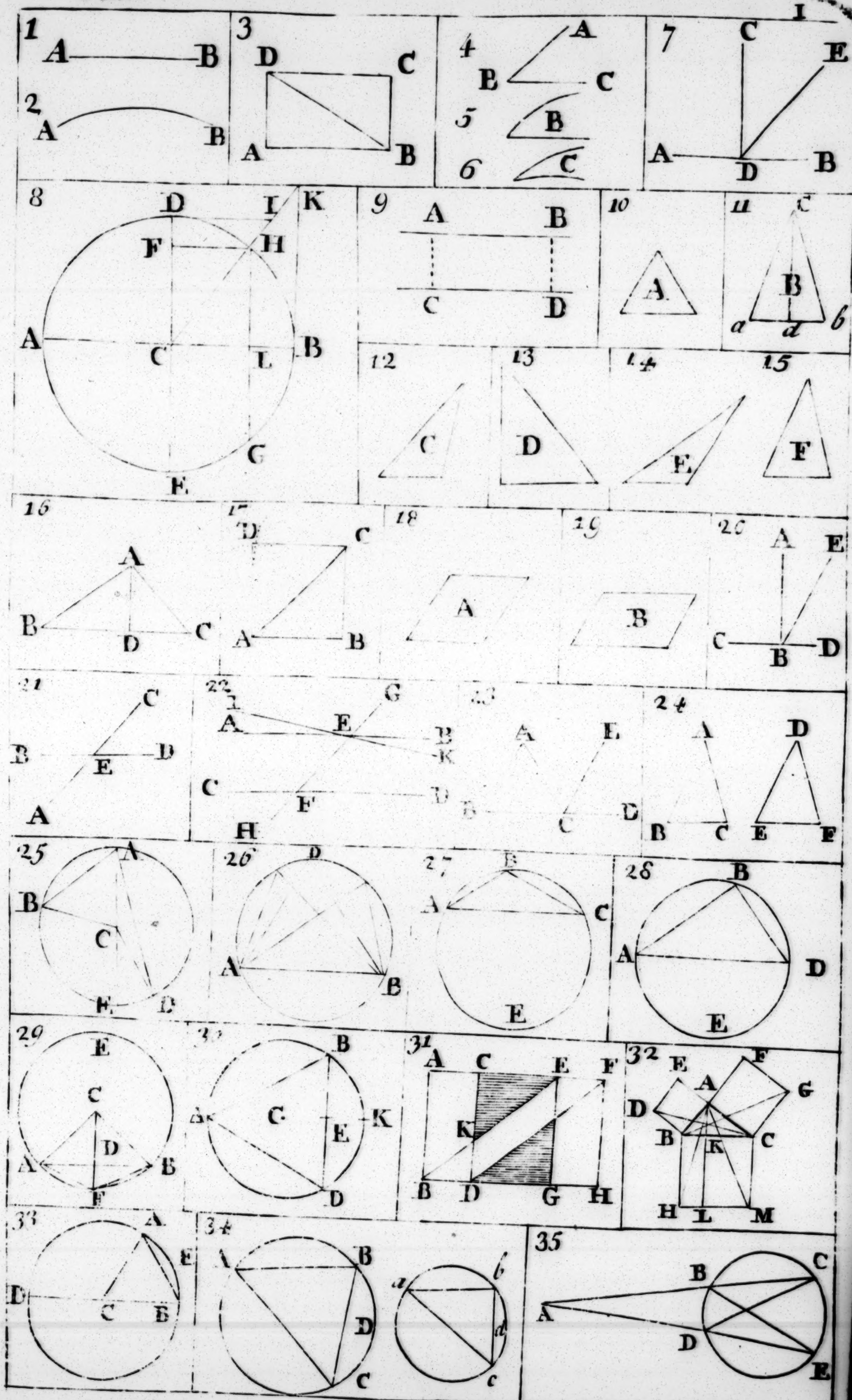
let

let a true meridian Line be drawn of about 3 Inches long, by which write *True Meridian.* Let a Scale be drawn, or it is ſufficient to expreſs the Number of Perches to an Inch, the Map was laid down by. Draw a Reference Table of three, or, if Occaſion be, of four or more Columns: In the firſt inſert the Number of the Field or Holding: In the next its Name, and by whom occupied: In the third the Quantity of Acres, Roods and Perches it contains: If you have unprofitable Land, as Bog or Mountain, let the Quantity be inſerted in the fourth Column; and, if it be required, you may make another Column for Statute Meaſure, and then the Map is compleated.

It has been uſual with Writers on this Subject to treat on the ſeveral Colours fit for Maps, and of the Manner of Waſhing, Grinding and Mixing them; but as ſuch may be had at many Druggiſts Shops ready made up in Shells, much better than I can pretend to adviſe, I refer ſuch as want, to apply to them.

FINIS.

Plate 1.



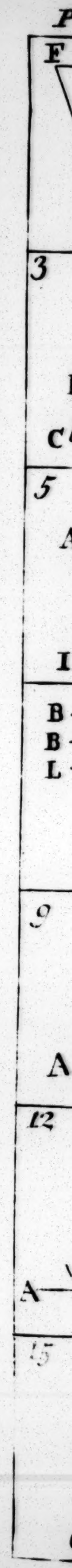
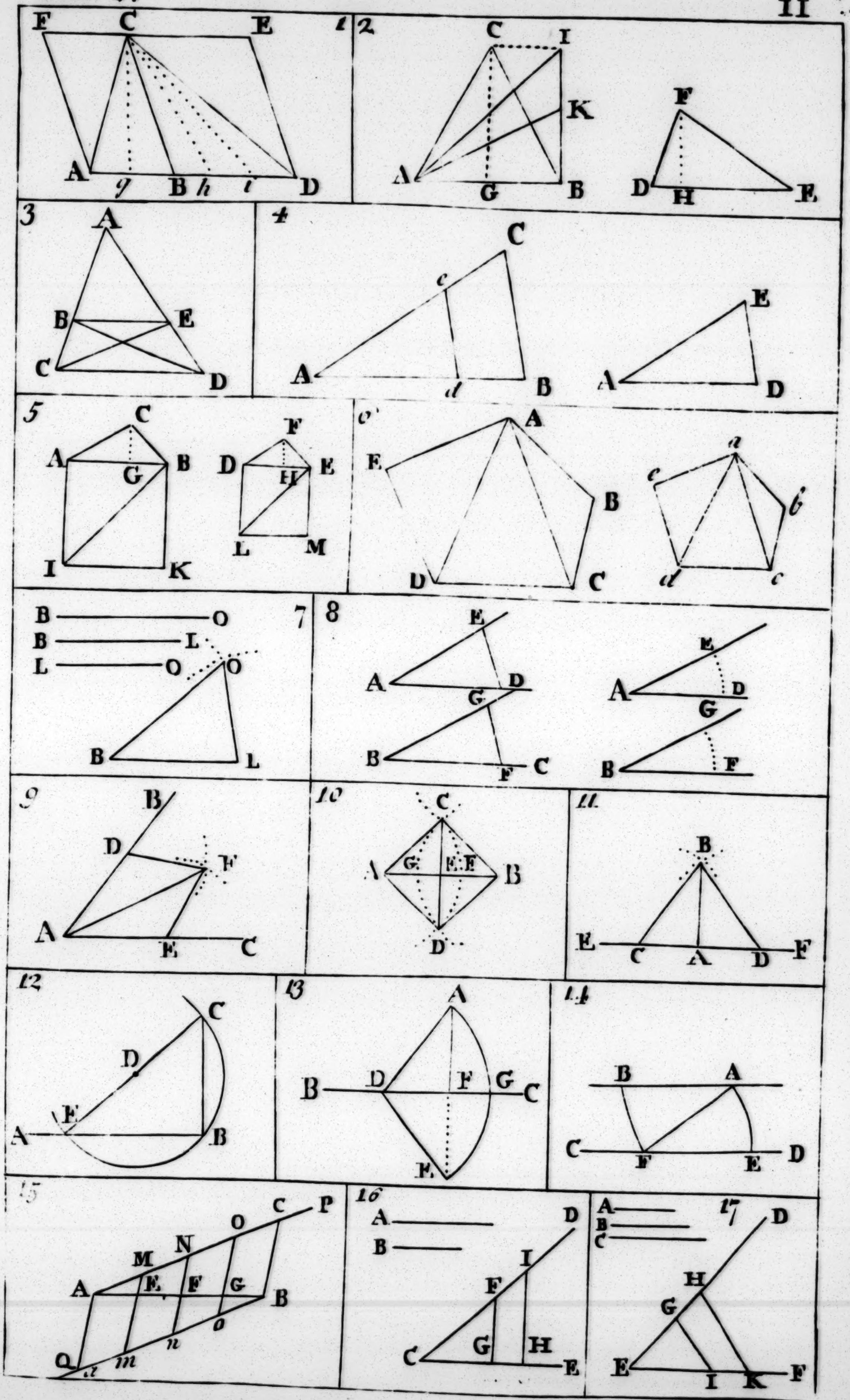


Plate 2.

II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2 C I K A G B F D H E
3 A B E C D
4 C c A d B E A D
5 C A G B I K F D H E L M
6 A E B D C a e b d c
7 B O B L L O O B L
8 E A D G B F C E A D G B F
9 B D F A E C
10 C A G E F B D
11 B E C A D F
12 C D E A B
13 A B D F G C E
14 B A C F E D
15 C P O N M A E F G B o n m Q a
16 A B D I F G H C E
17 A B C D H G E I K F

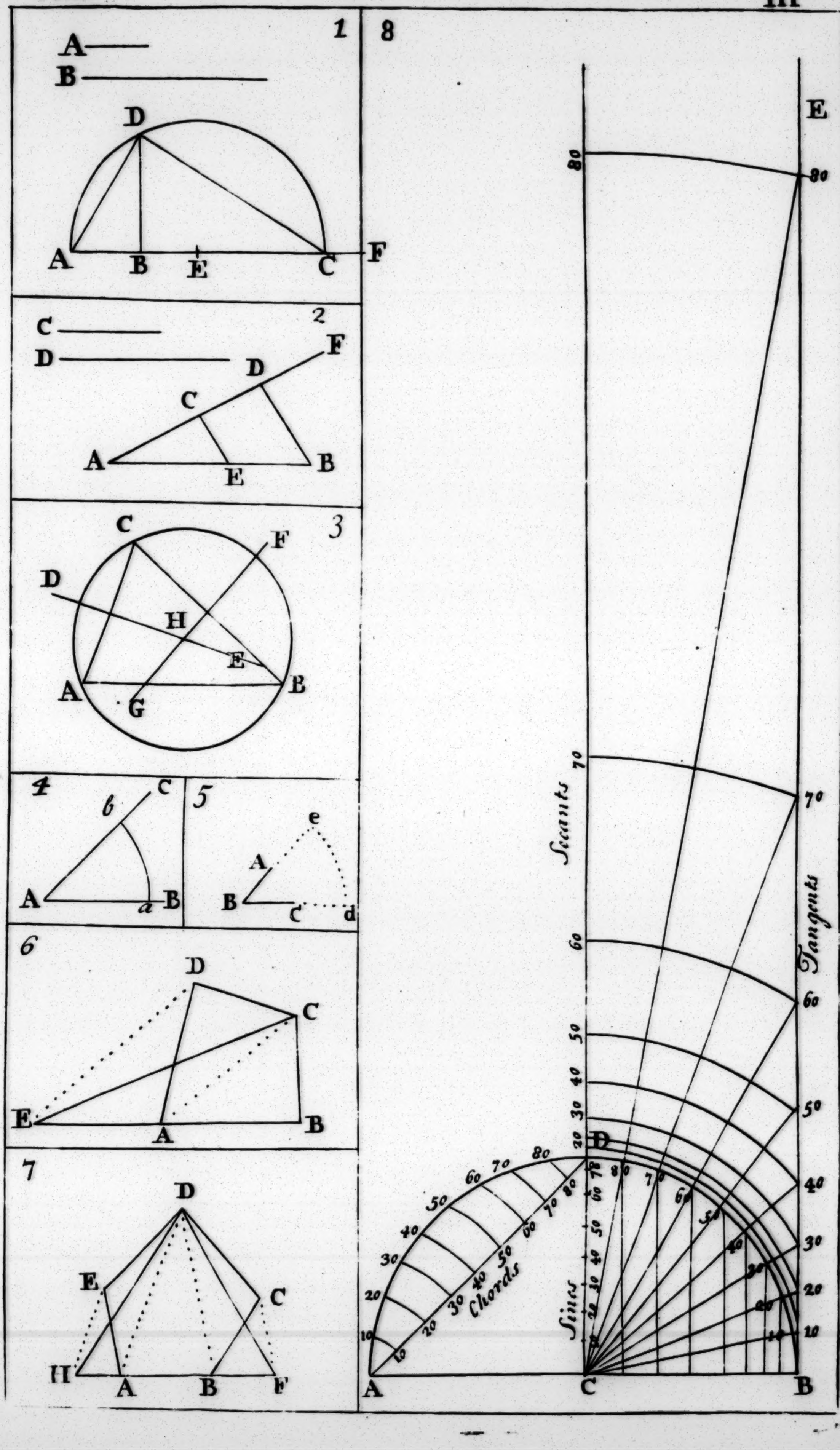
A

6

E

7

Plate .. III



Plate

The Proportions for the Solution of the 6 Cases of Right Angled Plane Triangles

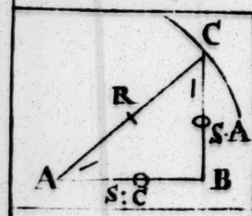
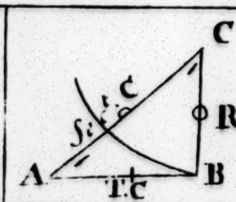
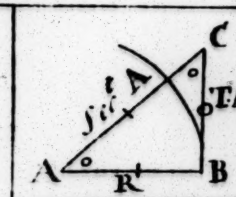
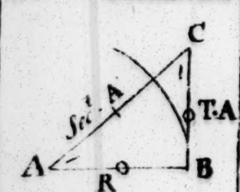
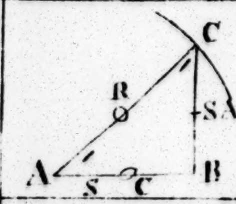
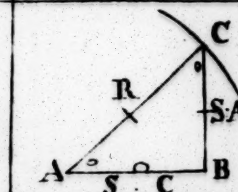
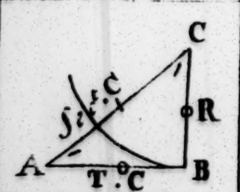
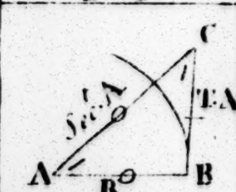
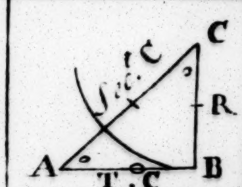
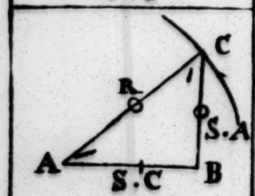
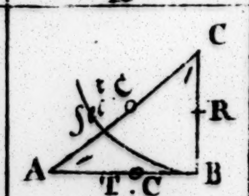
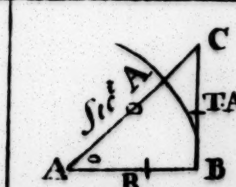
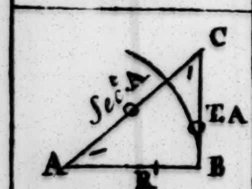
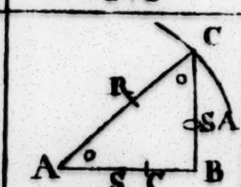
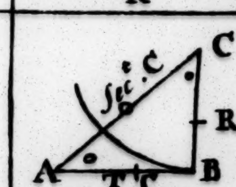
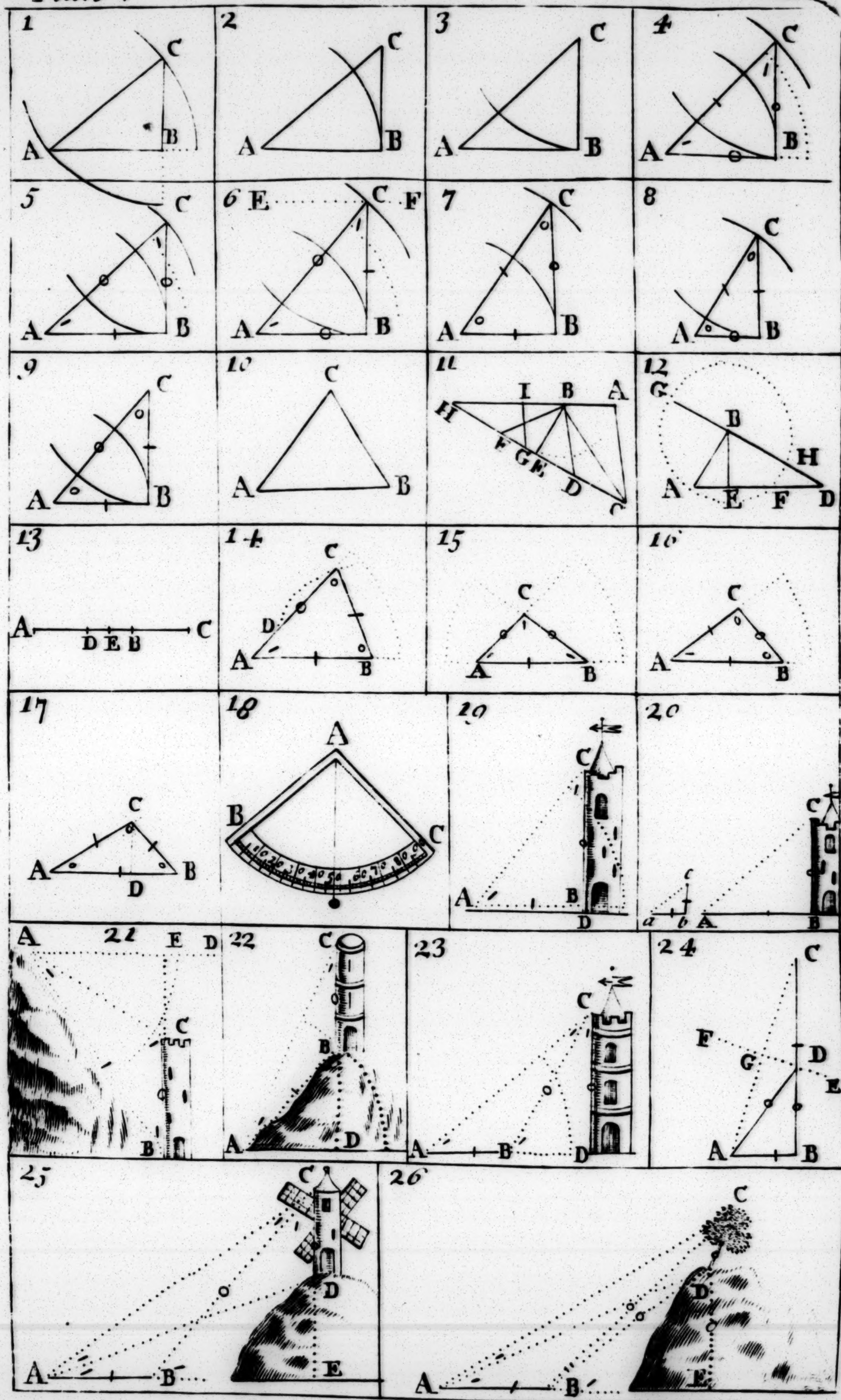
	Case 1 R : AC :: S.A : BC R : AC :: S.C : AB		T.C : AB :: R : BC T.C : AB :: Sec^t C : AC		AB : R :: AC : Sec^t A R : AB :: T.A : BC
	Sec^t A : AC :: R : AB Sec^t A : AC :: TA : BC		Case 3 S.A : BC :: R : AC S.A : BC :: S.C : AB		Case 5 AC : R :: BC : S.A R : AC :: S.C : AB
	Sec^t C : AC :: R : BC Sec^t C : AC :: T.C : AB		T.A : BC :: R : AB T.A : BC :: Sec^t A : AC		BC : R :: AC : Sec^t C R : BC :: T.C : AB
	Case 2 S.C : AB :: R : AC S.C : AB :: S.A : BC		R : BC :: T.C : AB R : BC :: Sec^t C : AC		Case 6 AB : R :: BC : T.A R : AB :: Sec^t. A : AC
	R : AB :: T.A : BC R : AB :: Sec^t A : AC		Case 4 AC : R :: AB : S.C R : AC :: S.A : BC		BC : R :: AB : T.C R : BC :: Sec^t. C : AC

Plate V


1 A B C
2 A B C
3 A B C
4 A B C
5 A B C
6 E C F A B
7 A B C
8 A B C
9 A B C
10 A B C
11 H I B A F G E D C
12 G B H A E F D
13 A D E B C
14 A D B C
15 A B C
16 A B C
17 A B C D
18 A B C
19 A B C D
20 a b c A B C
21 A E D C B
22 C B A D
23 C A B D
24 C F G D E A B
25 C D A B E
26 C D A B E

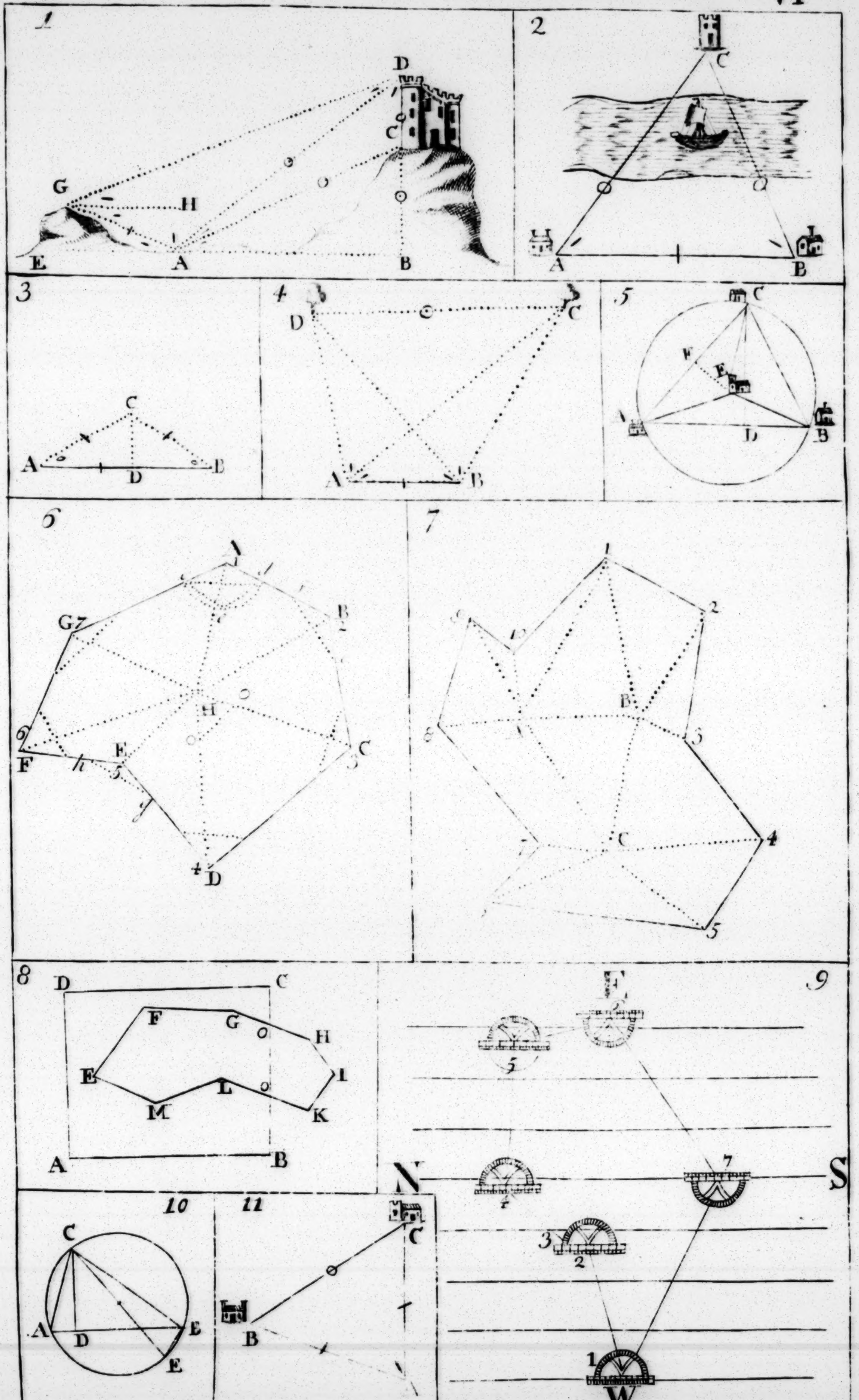
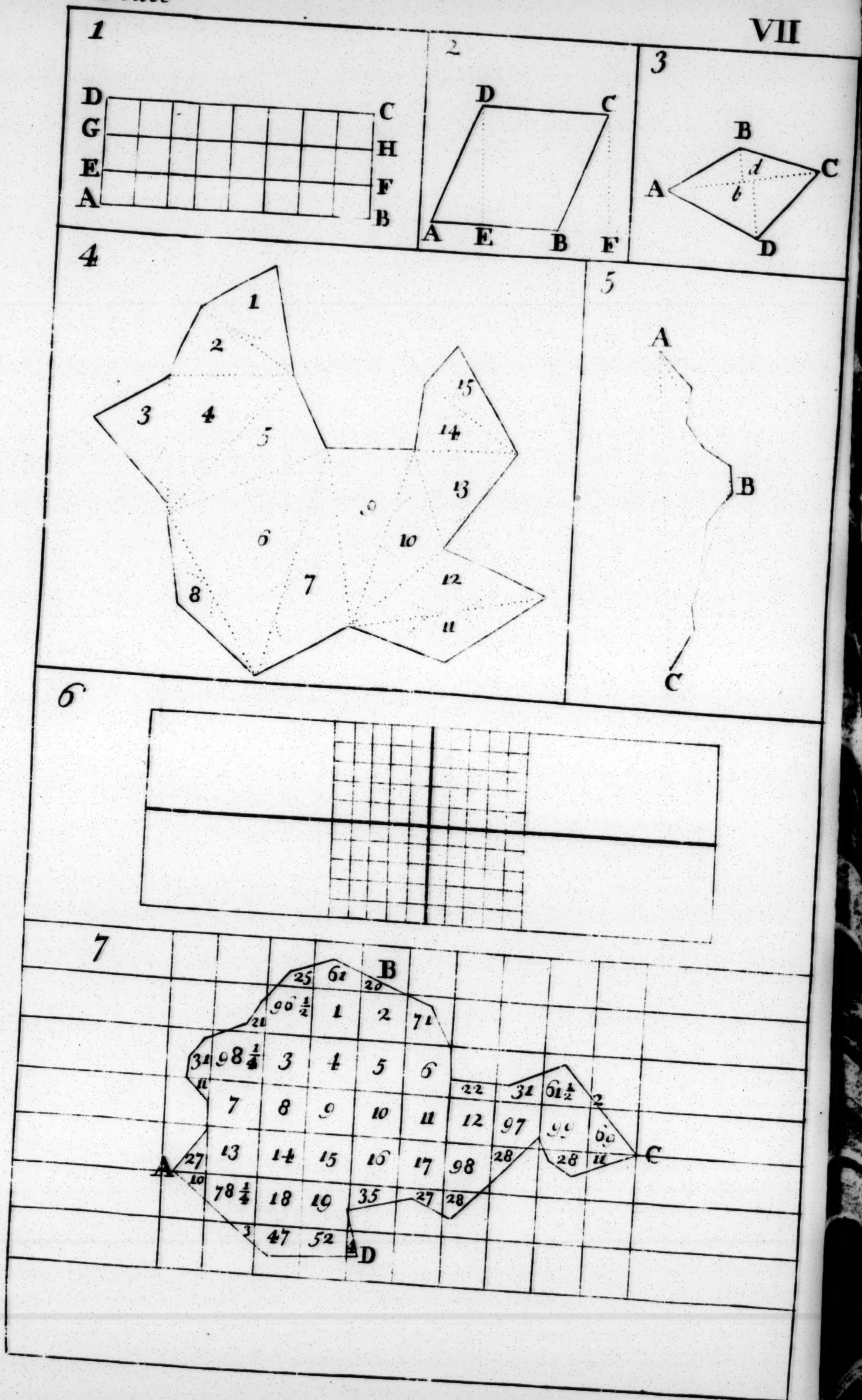

1
D
C
G
H
E
A
B
2
C
A
B
3
C
A
D
B
4
D
C
A
B
5
C
F
E
A
D
B
6
A
B
G
H
F
E
C
D
7
1
2
B
3
C
4
5
8
D
C
F
G
H
E
I
L
M
K
A
B
9
E
N
S
W
10
C
A
D
E
E
11
C
B

Plate VII



1
D
G
E
A
C
H
F
B
2
D
C
A
E
B
F
3
B
A
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4
1
2
3
4
5
6
7
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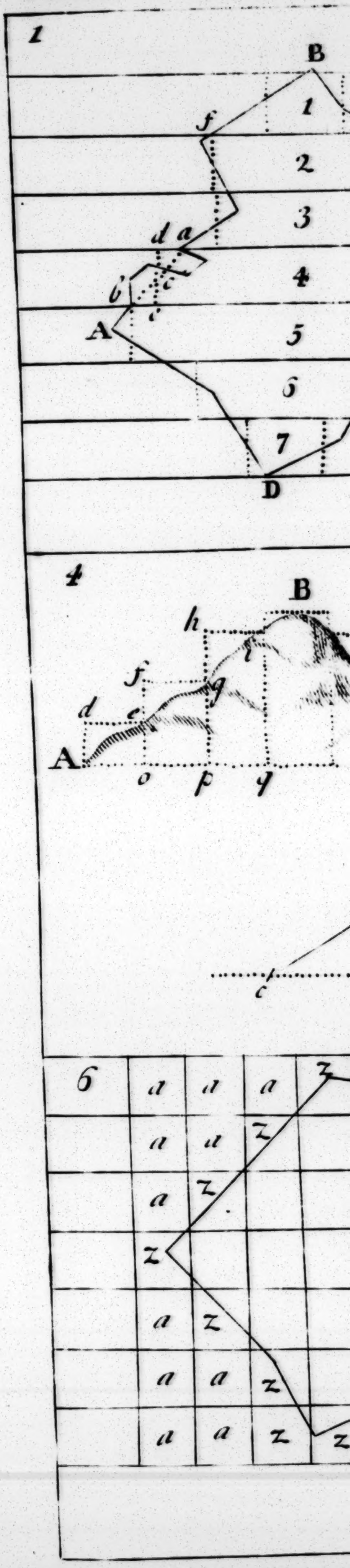
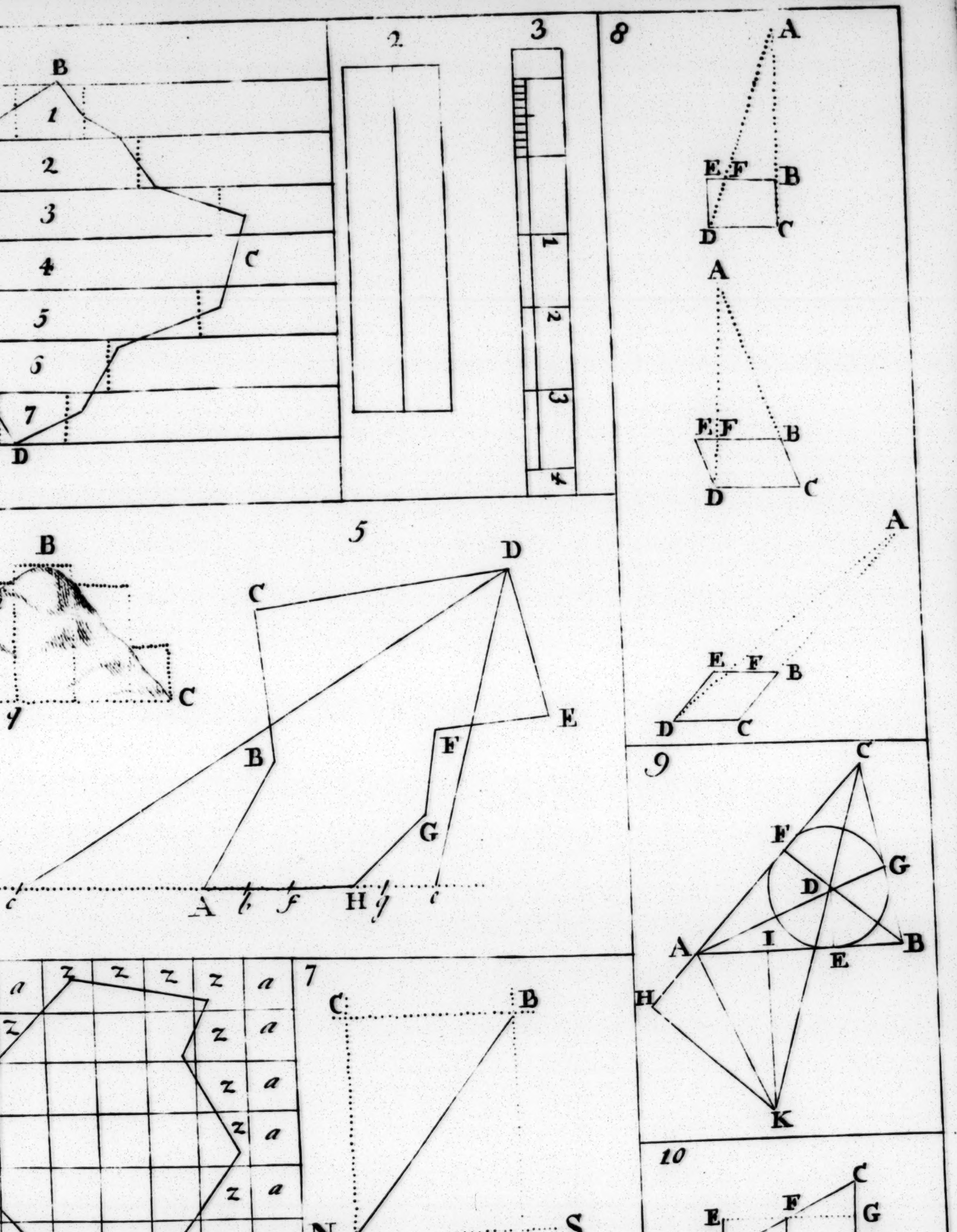
12
13
14
15
5
A
B
C
6
7
B
A
C
D

Plate




2
3
8
5
7
9
10

Plate

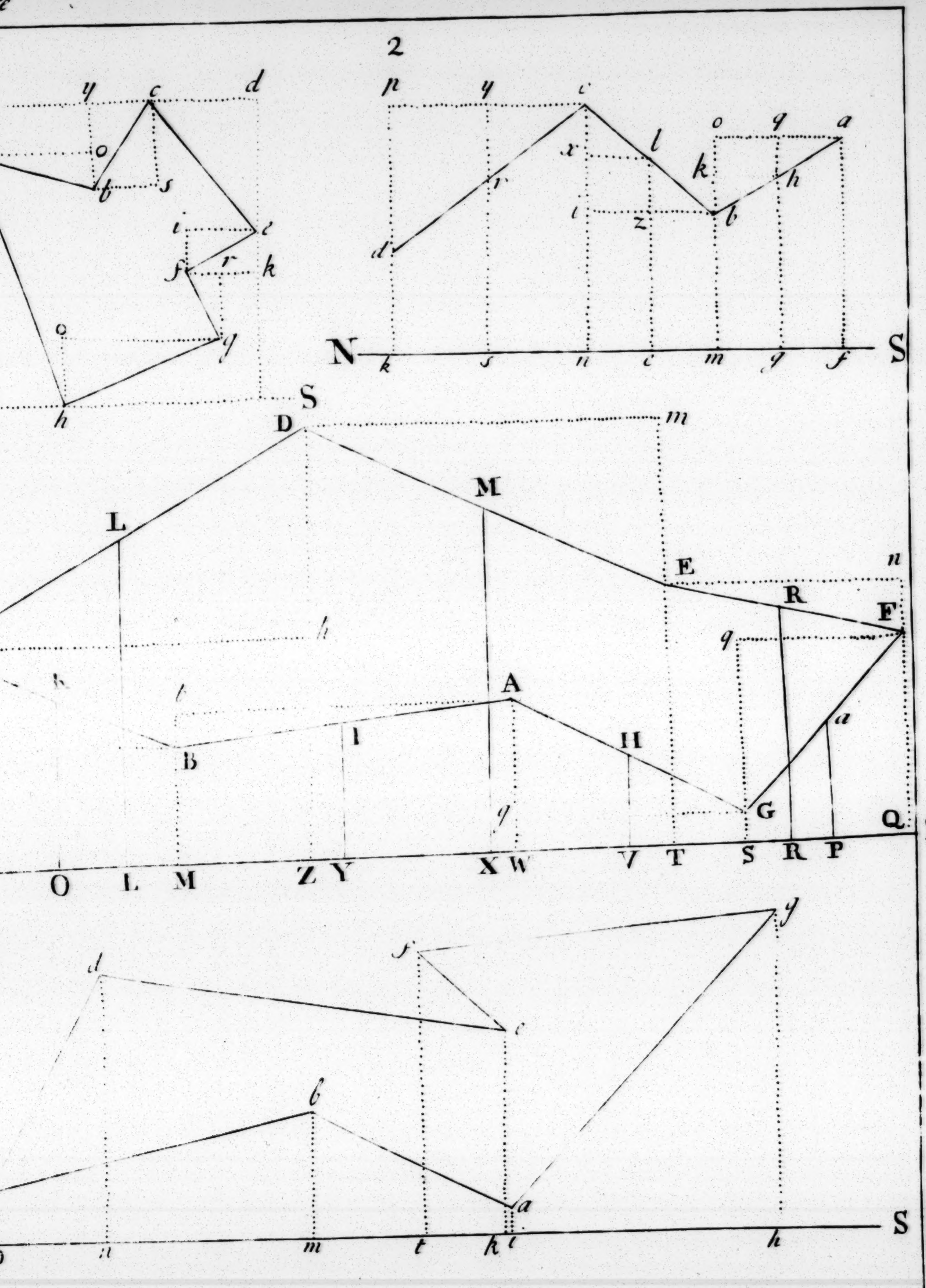
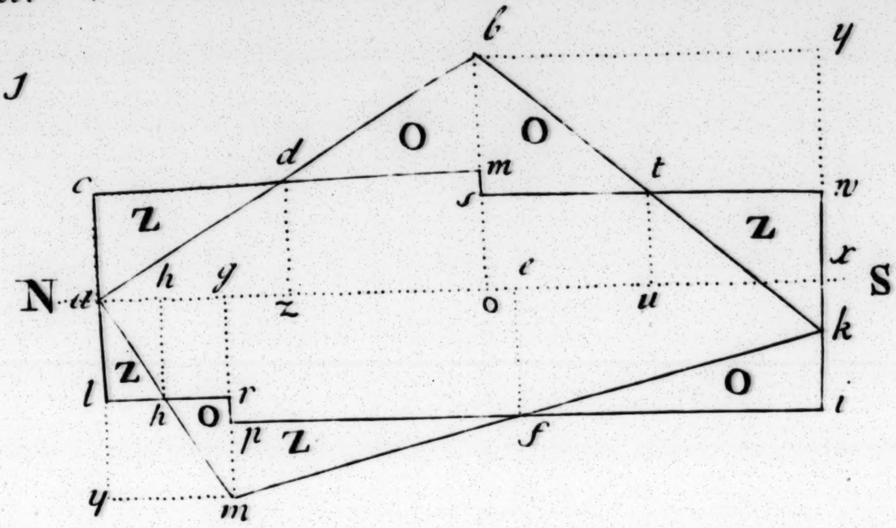

2
p y c d
x l o q a
k h
i z b
d r
N k s n c m g f S
S
D m
M
L
E n
R F
h q
A
a
I H
B
g
G Q S
O L M Z Y X W V T S R P
g
f
d
c
b
a
n m t k i h S

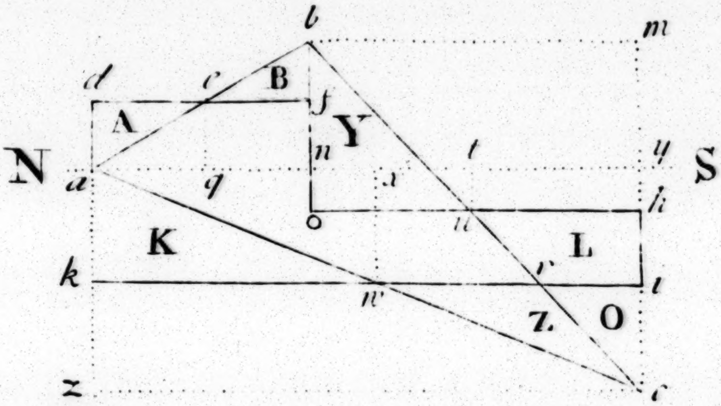
Plate X

1



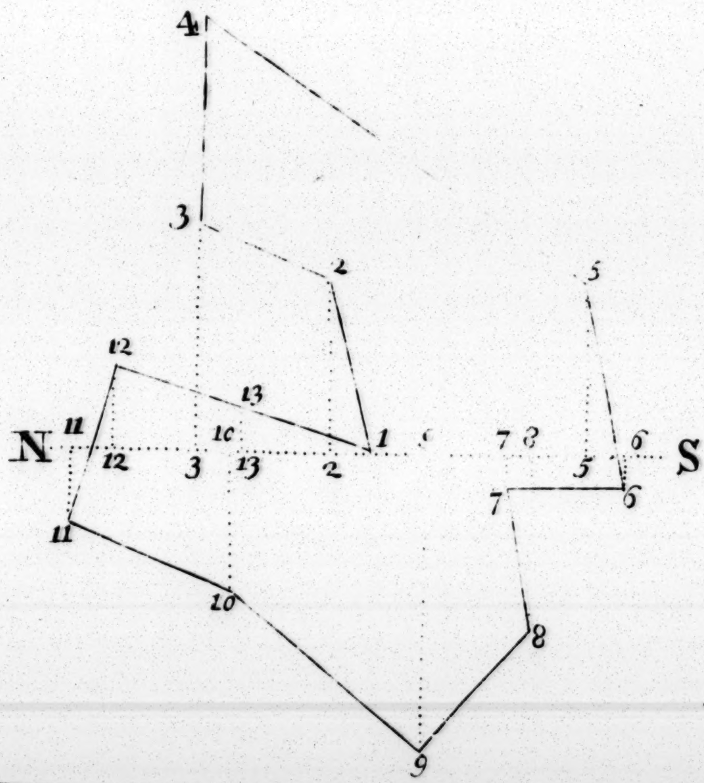
b
y
d
O
O
m
t
c
s
w
Z
Z
x
N
a
h
g
e
S
z
o
u
k
Z
l
r
O
h
O
p
Z
f
i
y
m

2



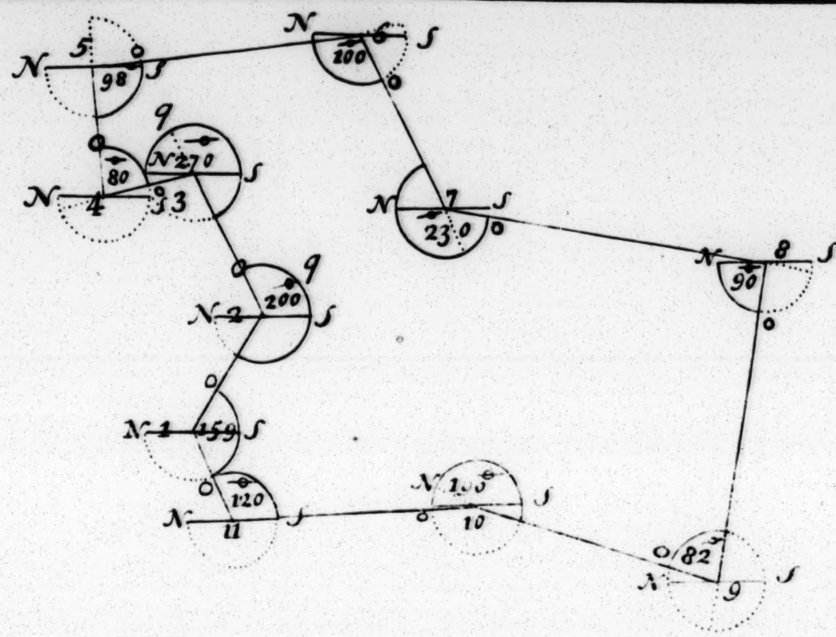
b
m
d
c
B
f
A
Y
N
n
t
y
S
a
q
x
h
o
u
K
L
r
k
w
i
Z
O
z
e

3

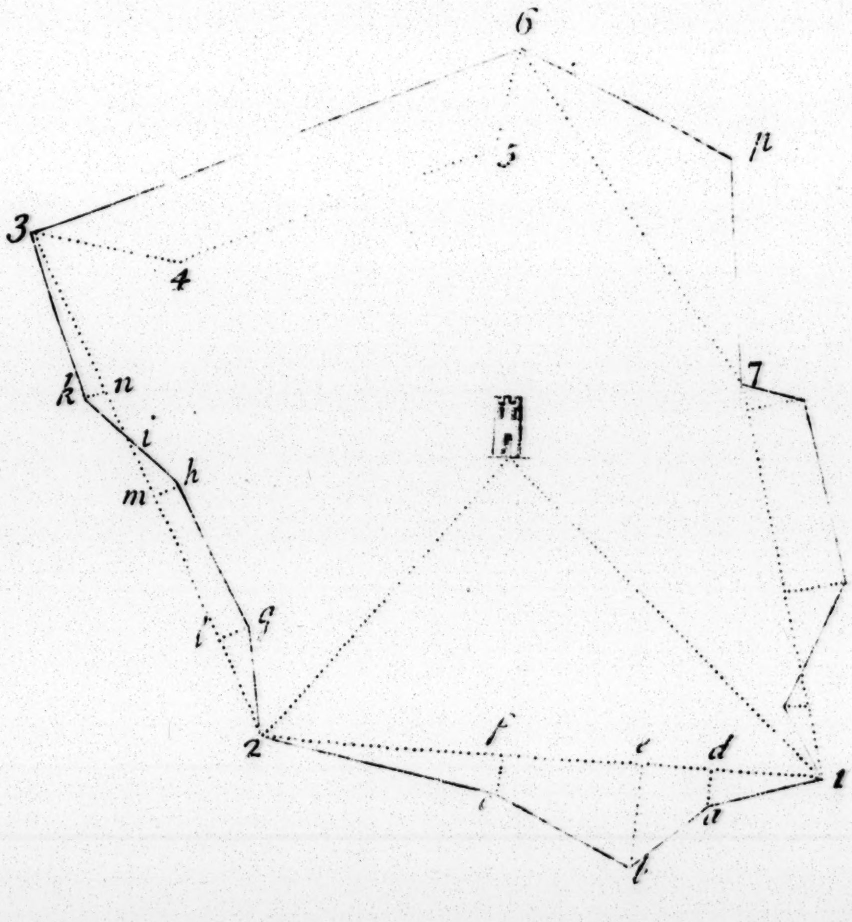


4
3
2
5
12
13
N
11
10
1
7
8
6
S
12
3
13
2
5
7
6
11
10
8
9

1


N
5
98
S
N
6
100
S
9
80
N 270
S
N
4
3
3
N
7
S
23.0
N
8
S
90
9
N 2
200
S
N 1
159
S
N
155
S
120
N
11
S
10
82
N
9
S

2

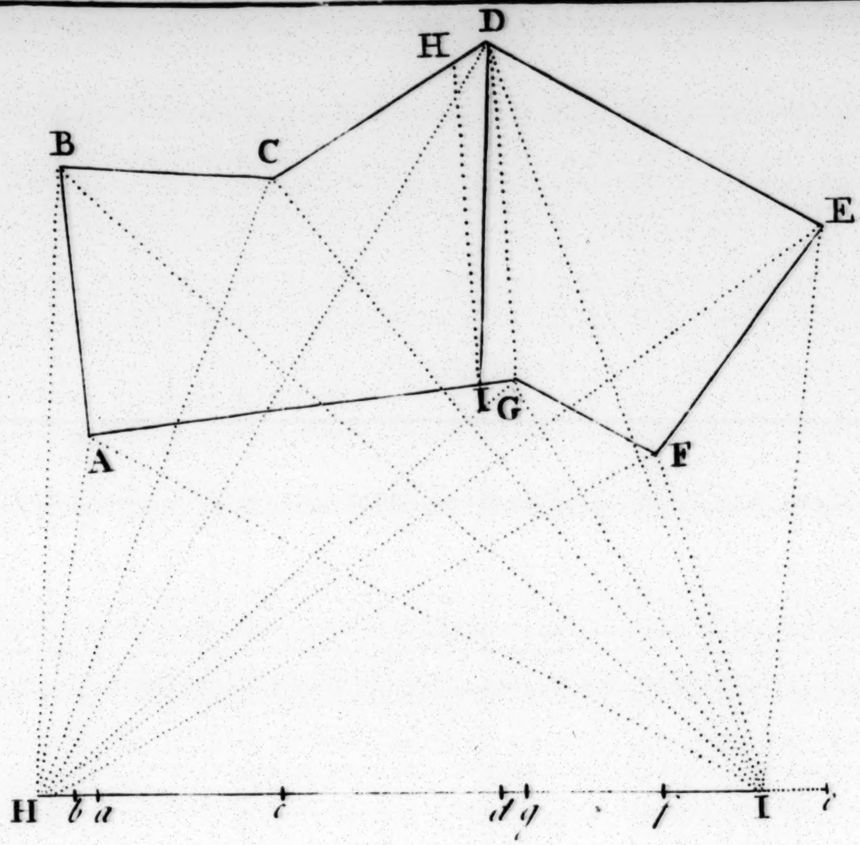

6
5
p
3
4
7
k
n
i
h
m
g
l
2
f
e
d
1
c
a
b

Plat

A

Plate XII

1



2